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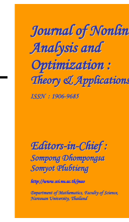

PICARD AND ADOMIAN DECOMPOSITION METHODS FOR A COUPLED SYSTEM OF QUADRATIC INTEGRAL EQUATIONS OF FRACTIONAL ORDER

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ABSTRACT. The comparison between the classical method of successive approximations (Picard) method and Adomian decomposition method was studied in many papers for example ([14] and [37]).
In this paper we are concerning with two analytical methods; the classical method of successive approximations (Picard)[18] and Adomian decomposition methods ([1]-[6], [16] and [17]) for a coupled system of quadratic integral equations of fractional order. Also, the existence and uniqueness of the solution and the convergence will be discussed for each method and some examples will be studied.

KEYWORDS : Coupled systems; Quadratic integral equation; Picard method; Adomian method; Continuous unique solution; Fractional-order integration; Convergence analysis; Error analysis.
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1. INTRODUCTION

Quadratic integral equations (QIEs) are often applicable in the theory of radiative transfer, kinetic theory of gases, in the theory of neutron transport and in the traffic theory. The quadratic integral equations can be very often encountered in many applications.
The quadratic integral equations have been studied in several papers and monographs (see for examples [8]-[12] and [20]-[26]).
The authors [27] proved the existence and the uniqueness of continuous solution for the quadratic integral equation

$$x(t) = a(t) \ + \ g(t, x(t)) \int_0^t f(s, x(s)) \, ds$$

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by using the principle of contraction mapping and comparing the two analytical methods; the classical method of successive approximations (Picard)[18] which consists the construction of a sequence of functions such that the limit of this sequence of functions in the sense of uniform convergence is the solution of the quadratic integral equation, and Adomian decomposition method which gives the solution as a series see([1]-[6], [16] and [17]). Also, from the results of the examples the authors deduced that Picard method gives more accurate solution than ADM.

Systems occur in various problems of applied nature, for instance, see ([27]-[15], [29]-[31]). Recently, Su [36] discussed a two-point boundary value problem for a coupled system of fractional differential equations. Gafiychuk et al. [31] analyzed the solutions of coupled nonlinear fractional reaction-diffusion equations.
The coupled systems have been studied in many papers; see [27], [36] and [37].

This paper deals with the coupled system of quadratic integral equations of fractional order

$$\begin{aligned} x(t) &= a_1(t) + g_1(t,y(t)) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1(s,y(s))\, ds,\ t\in[0,1], \\ y(t) &= a_2(t) + g_2(t,x(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2(s,x(s))\, ds,\ t\in[0,1], \end{aligned} \tag{1.1}$$

where $\alpha,\ \beta > 0$.
and comparing the results obtained from the two methods; Picard and Adomian decomposition methods. Also, some examples will be studied.

Now, the definition of the fractional-order integral operator is given by:

Definition 1.1. Let β be a positive real number, the fractional-order integral of order β of the function f is defined on the interval $[a,b]$ by (see [32], [33], [34] and [35])

$$I_a^\beta f(t) = \int_a^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f(s)\, ds,$$

and when $a=0$, we have $I^\beta f(t) = I_0^\beta f(t)$.

For further properties of fractional-order integral operator (see [32]-[35]) for example.

2. METHOD OF SUCCESSIVE APPROXIMATIONS (PICARD METHOD)

Now, the coupled system (1.1) will be investigated under the assumptions:

(i) $a_i : I \to R_+ = [0,+\infty),\ i=1,2$ is continuous on I where $I=[0,1]$;

(ii) $f_i,\ g_i : I\times D \subset R_+ \to R_+,\ i=1,2$ are continuous and there exist positive constants M_i and $N_i,\ i=1,2$ such that $|g_i(t,x)| \le M_i$ and $|f_i(t,x)| \le N_i$ on D;

(iii) $f_i,\ g_i,\ i=1,2$ satisfy Lipschitz condition with Lipschitz constants L_i and K_i such that,

$$\begin{aligned} |g_i(t,x) - g_i(t,y)| &\le L_i|x-y|, \\ |f_i(t,x) - f_i(t,y)| &\le K_i|x-y|. \end{aligned}$$

Let $C = C(I)$ be the space of all real valued functions which are continuous on I.
Define the operators T_1, T_2 by

$$T_1 y(t) = a_1(t) + g_1(t, y(t)) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1(s, y(s))\, ds,\ t \in I$$

$$T_2 x(t) = a_2(t) + g_2(t, x(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2(s, x(s))\, ds,\ t \in I,$$

where $\alpha,\ \beta > 0$.
Then the coupled (1.1) may be written as:

$$x(t) = T_1 y(t)$$
$$y(t) = T_2 x(t).$$

Define the operator T by

$$T(x, y)(t) = (T_1 y(t),\ T_2 x(t)).$$

Theorem 2.1. *Let the assumptions (i)-(iii) be satisfied. If* $(M_1K_1 + N_1L_1)(M_2K_2 + N_2L_2) < 1$, *then the coupled system of quadratic integral equations of fractional order (1.1) has a unique positive solution* $(x, y) \in C \times C$.

Proof. It is clear that the operators $T_1,\ T_2$ map C into C.
Applying Picard method to the coupled system of quadratic integral equation (1.1), the solution is constructed by the sequences

$$\begin{aligned} x_n(t) &= a_1(t) + g_1(t, y_{n-1}(t)) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1(s, y_{n-1}(s))\, ds,\ n = 1, 2, \ldots, \\ x_0(t) &= a_1(t) \\ y_n(t) &= a_2(t) + g_2(t, x_{n-1}(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2(s, x_{n-1}(s))\, ds,\ n = 1, 2, \ldots, \\ y_0(t) &= a_2(t). \end{aligned} \tag{2.1}$$

All the functions $x_n(t)$ and $y_n(t)$ are continuous functions. Also, $x_n(t)$ and $y_n(t)$ can be written as a sum of successive differences:

$$x_n = x_0 + \sum_{j=1}^{n} (x_j - x_{j-1}),$$

$$y_n = y_0 + \sum_{j=1}^{n} (y_j - y_{j-1}).$$

This means that convergence of the two sequences $\{x_n\}$ and $\{y_n\}$ is equivalent to convergence of the two infinite series $\sum(x_j - x_{j-1})$, $\sum(y_j - y_{j-1})$ and the solution will be
$u(t) = (x(t), y(t))$, where

$$x(t) = \lim_{n \to \infty} x_n(t),$$

$$y(t) = \lim_{n \to \infty} y_n(t),$$

i.e. if the two infinite series $\sum(x_j - x_{j-1})$, $\sum(y_j - y_{j-1})$ converge, then the two sequence $\{x_n(t)\}$, $\{y_n(t)\}$ will converge to $x(t)$ and $y(t)$ respectively. To prove the uniform convergence of $\{x_n(t)\}$ and $\{y_n(t)\}$ we shall consider the two associated series

$$\sum_{n=1}^{\infty}[x_n(t) - x_{n-1}(t)],$$
$$\sum_{n=1}^{\infty}[y_n(t) - y_{n-1}(t)].$$

From (2.1) for $n = 1$, we get

$$x_1(t) - x_0(t) = g_2(t, y_0(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2(s, y_0(s))\, ds$$
$$y_1(t) - y_0(t) = g_1(t, x_0(t)) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1(s, x_0(s))\, ds$$

and

$$|\, x_1(t) - x_0(t)\, | \le M_2N_2 \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)}\, ds \le M_2N_2\, \frac{t^\beta}{\Gamma(\beta+1)}.$$

Also,

$$|\, y_1(t) - y_0(t)\, | \le M_1N_1\, \frac{t^\alpha}{\Gamma(\alpha+1)}. \tag{2.2}$$

Now, we shall obtain an estimate for $x_n(t) - x_{n-1}(t),\ n \ge 2$

$$\begin{aligned}
x_n(t) - x_{n-1}(t) \quad &\le \quad g_2(t, y_{n-1}(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2(s, y_{n-1}(s))\, ds \\
&- \quad g_2(t, y_{n-2}(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2(s, y_{n-2}(s))\, ds \\
&+ \quad g_2(t, y_{n-1}(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2(s, y_{n-2}(s))\, ds \\
&- \quad g_2(t, y_{n-1}(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2(s, y_{n-2}(s))\, ds \\
&\le \quad g_2(t, y_{n-1}(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} [f_2(s, y_{n-1}(s)) - f_2(s, y_{n-2}(s))]\, ds \\
&+ \quad [\, g_2(t, y_{n-1}(t)) - g_2(t, y_{n-2}(t))\,] \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_2(s, y_{n-2}(s))\, ds,
\end{aligned}$$

using assumptions (ii) and (iii), we get

$$\begin{aligned}
|x_n(t) - x_{n-1}(t)| \le M_2K_2 &\int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} |y_{n-1}(s) - y_{n-2}(s)|\, ds \\
&+ N_2L_2|y_{n-1}(t) - y_{n-2}(t)| \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)}\, ds.
\end{aligned}$$

Putting $n = 2$, then using (2.2) we get

$$\begin{aligned}
|x_2(t) - x_1(t)| \quad &\le \quad M_2K_2 \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} |y_1(s) - y_0(s)\,|\, ds \\
&+ \quad N_2\ L_2|\ y_1(t) - y_0(t)\ | \frac{t^\beta}{\Gamma(\beta+1)} \\
|x_2(t) - x_1(t)| \quad &\le \quad M_2M_1N_1K_2 \frac{t^{\alpha+\beta}}{\Gamma(\alpha+1)\Gamma(\beta+\alpha+1)} + M_1N_1N_2L_2 \frac{t^{\alpha+\beta}}{\Gamma(\alpha+1).\Gamma(\beta+1)} \\
&\le \quad M_1N_1(M_2K_2 + N_2L_2)\ t^{\alpha+\beta}.
\end{aligned}$$

By the same way we can prove that:

$$|y_2(t) - y_1(t)| \quad \leq \quad M_2 N_2 (M_1 K_1 + N_1 L_1)\; t^{\alpha+\beta}$$

using the above estimate we get

$$\begin{aligned} |x_3(t) - x_2(t)| &\leq M_2 K_2 \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)}\, |y_2(s) - y_1(s)|\; ds \\ &+ N_2 L_2\; |y_2(t) - y_1(t)| \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)}\; ds \\ &\leq M_2 N_2 (M_1 K_1 + N_1 L_1)(M_2 K_2 + N_2 L_2)\; t^{2\alpha+\beta}. \end{aligned}$$

by a similar way as done before we have the following:

$$|y_3(t) - y_2(t)| \leq M_2 N_2 (M_1 K_1 + N_1 L_1)(M_2 K_2 + N_2 L_2)\; t^{\alpha+2\beta}$$

$$|x_4(t) - x_3(t)| \leq M_2 N_2 (M_1 K_1 + N_1 L_1)^2 (M_2 K_2 + N_2 L_2)\; t^{2\alpha+2\beta}$$

$$|y_4(t) - y_3(t)| \leq M_1 N_1 (M_1 K_1 + N_1 L_1)^2 (M_2 K_2 + N_2 L_2)\; t^{2\alpha+2\beta}$$

$$|x_5(t) - x_4(t)| \leq M_1 N_1 (M_1 K_1 + N_1 L_1)^2 (M_2 K_2 + N_2 L_2)^2\; t^{3\alpha+2\beta}$$

Repeating this technique, we obtain the general estimate for the terms of the series:

$$|x_n(t) - x_{n-1}(t)| \;\leq\; \begin{cases} M_2 N_2 (M_1 K_1 + N_1 L_1)^{\frac{n}{2}} (M_2 K_2 + N_2 L_2)^{\frac{n}{2}-1} & \text{for } n \text{ even} \\ \\ M_1 N_1 (M_1 K_1 + N_1 L_1)^{\frac{n-1}{2}} (M_2 K_2 + N_2 L_2)^{\frac{n-1}{2}} & \text{for } n \text{ odd} \end{cases}$$

and

$$|y_n(t) - y_{n-1}(t)| \;\leq\; \begin{cases} M_1 N_1 (M_1 K_1 + N_1 L_1)^{\frac{n}{2}} (M_2 K_2 + N_2 L_2)^{\frac{n}{2}-1} & \text{for } n \text{ even} \\ \\ M_2 N_2 (M_1 K_1 + N_1 L_1)^{\frac{n-1}{2}} (M_2 K_2 + N_2 L_2)^{\frac{n-1}{2}} & \text{for } n \text{ odd} \end{cases}$$

Since $(M_1 K_1 + N_1 L_1)(M_2 K_2 + N_2 L_2) < 1$, then the uniform convergence of

$$\sum_{n=1}^{\infty} [x_n(t) - x_{n-1}(t)]$$

and

$$\sum_{n=1}^{\infty} [y_n(t) - y_{n-1}(t)]$$

is proved and so the sequences $\{x_n(t)\}$ and $\{y_n(t)\}$ are uniformly convergent. Since $f_i(t,x)$ and $g_i(t,x)$ are continuous in the second argument then

$$\begin{aligned} x(t) &= a_1(t) + \lim_{n\to\infty} g_1(t, y_{n-1}(t)) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\; f_1(s, y_{n-1}(s))\; ds \\ &= a_1(t) + g_1(t, y(t)) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\; f_1(s, y(s))\; ds. \end{aligned}$$

and

$$\begin{aligned} y(t) &= a_2(t) + \lim_{n\to\infty} g_2(t, x_{n-1}(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)}\; f_2(s, x_{n-1}(s))\; ds \\ &= a_2(t) + g_2(t, x(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)}\; f_2(s, x(s))\; ds. \end{aligned}$$

Therefore, the sequence $\{u_n(t)\}$ which is defined by $u_n(t) = (x_n(t),\; y_n(t))$ is uniformly convergent. Thus, the existence of a solution is proved.

To prove the uniqueness, let $\tilde{u}(t) = (\tilde{x}, \tilde{y})(t)$ be a continuous solution of (1.1). Then

$$\begin{aligned}
\tilde{x}(t) &= a_1(t) + g_1(t, \tilde{y}(t)) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1(s, \tilde{y}(s))\, ds,\ t \in [0,1], \\
\tilde{y}(t) &= a_2(t) + g_1(t, \tilde{x}(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2(s, \tilde{x}(s))\, ds,\ t \in [0,1],
\end{aligned}$$

and

$$\begin{aligned}
|\tilde{u}(t) - u_n(t)| &= |(\tilde{x}(t),\ \tilde{y}(t)) - (x_n(t),\ y_n(t))| \\
&= |(\tilde{x}(t) - x_n(t),\ \tilde{y}(t) - y_n(t))| \\
&\le \sup_{t \in I} |(\tilde{x}(t) - x_n(t),\ \tilde{y}(t) - y_n(t))| \\
&\le ||(\tilde{x}(t) - x_n(t),\ \tilde{y}(t) - y_n(t))|| \\
&\le ||\tilde{x}(t) - x_n(t)|| + ||\tilde{y}(t)) - y_n(t)||,
\end{aligned}$$

by a simple calculations we get

$$\lim_{n\to\infty} x_n(t) = x(t) = \tilde{x}(t),$$

$$\lim_{n\to\infty} y_n(t) = y(t) = \tilde{y}(t).$$

Therefore

$$\lim_{n\to\infty} u_n(t) = u(t) = \tilde{u}(t).$$

Which completes the proof. □

Corollary 2.1. *Let the assumptions of Theorem 2.1 be satisfied. If* $\alpha,\ \beta \to 1$, *then the coupled system of quadratic integral equation*

$$\begin{aligned}
x(t) &= a_1(t) + g_1(t, y(t)) \int_0^t f_1(s, y(s))\, ds \\
y(t) &= a_1(t) + g_2(t, x(t)) \int_0^t f_2(s, x(s))\, ds
\end{aligned}$$

has a unique continuous solution.

3. ADOMIAN DECOMPOSITION METHOD (ADM)

The Adomian decomposition method (ADM) is a non-numerical method for solving a wide variety of functional equations and usually gets the solution in a series form.

Since the beginning of the 1980s, Adomian ([1]-[6] and [16]-[17]) has presented and developed a so-called decomposition method for solving algebraic, differential, integro- differential, differential-delay, and partial differential equations. The solution is found as an infinite series which converges rapidly to accurate solutions. The method has many advantages over the classical techniques, mainly, it makes unnecessary the linearization, perturbation and other restrictive methods and assumptions which may change the problem being solved, sometimes seriously. In recent decades, there has been a great deal of interest in the Adomian decomposition method. The method was successfully applied to a large amount of applications in applied sciences. For more details about the method and its application, see ([1]-[6], [37] and [16]-[17]).

In this section, we shall study Adomian decomposition method (ADM) for the coupled system (1.1).
The solution algorithm of the coupled system (1.1) using ADM is,

$$x_0(t) = a_1(t),\ x_i(t) = A_{i-1}(t)\int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} B_{i-1}(s)\,ds,\ i \geq 1, \tag{3.1}$$

$$y_0(t) = a_2(t),\ y_i(t) = C_{i-1}(t)\int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} D_{i-1}(s)\,ds,\ i \geq 1, \tag{3.2}$$

where A_i, B_i, C_i and D_i are Adomian polynomials of the nonlinear terms

$g_1(t, y(t))$, $f_1(s, y(s))$, $g_2(t, x(t))$ and $f_2(s, x(s))$ respectively, which take the forms

$$\begin{aligned}
A_i &= \frac{1}{i!}\left[\frac{d^i}{d\lambda^i}\, g_1\left(t, \sum_{k=0}^{\infty}\lambda^k y_k\right)\right]_{\lambda=0},\\
B_i &= \frac{1}{i!}\left[\frac{d^i}{d\lambda^i}\, f_1\left(s, \sum_{k=0}^{\infty}\lambda^k y_k\right)\right]_{\lambda=0},\\
C_i &= \frac{1}{i!}\left[\frac{d^i}{d\lambda^i}\, g_2\left(t, \sum_{k=0}^{\infty}\lambda^k x_k\right)\right]_{\lambda=0},\\
D_i &= \frac{1}{i!}\left[\frac{d^i}{d\lambda^i}\, f_2\left(s, \sum_{k=0}^{\infty}\lambda^k x_k\right)\right]_{\lambda=0}.
\end{aligned}$$

Finally, the solution of the coupled system (1.1) will be

$$x(t) = \sum_{i=0}^{\infty} x_i(t) \text{ and } y(t) = \sum_{i=0}^{\infty} y_i(t) \tag{3.3}$$

4. CONVERGENCE ANALYSIS

4.1. **Existence and Uniqueness theorem.**

Theorem 4.1. *Let* $a_1(t), a_2(t) \in C(I)$. *If* $0 < R < 1$ *then the coupled system (1.1) has a unique solution* $X \in C^2(I)$, *where* $X = \begin{pmatrix} x \\ y \end{pmatrix}$, $R = \max\{r_1, r_2\}$, $r_1 = \frac{1}{\Gamma(\beta+1)}[L_2N_2 + K_2M_2], r_2 = \frac{1}{\Gamma(\alpha+1)}[L_1N_1 + K_1M_1]$.

Proof. The system (1.1):

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} g_2(t, x(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2(s, x(s))\,ds \\ g_1(t, y(t)) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1(s, y(s))\,ds \end{pmatrix}$$

can be written as,

$$X = G + DM,$$

where,

$$X = \begin{pmatrix} x \\ y \end{pmatrix}, G = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, D = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, M = \begin{pmatrix} g_2(t, x(t)) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2(s, x(s))\,ds \\ g_1(t, y(t)) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1(s, y(s))\,ds \end{pmatrix}$$

The mapping $F : E \to E$ is defined as,

$$FX = G + DM,$$

Let $X, U \in E$, then

$$FU = G + DN,$$

where,

$$U = \begin{pmatrix} u \\ v \end{pmatrix}, \ N = \begin{pmatrix} g_2\left(t, u(t)\right) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2\left(s, u\left(s\right)\right) ds \\ g_1\left(t, v(t)\right) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1\left(s, v\left(s\right)\right) ds \end{pmatrix}.$$

so,

$$\begin{aligned}
\|FX - FU\| &= \|D\| \, \|M - N\| \\
&= \left\| \begin{pmatrix} g_2\left(t, x(t)\right) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2\left(s, x\left(s\right)\right) ds - g_2\left(t, u(t)\right) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2\left(s, u\left(s\right)\right) ds \\ \\ g_1\left(t, y(t)\right) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1\left(s, y\left(s\right)\right) ds - g_1\left(t, v(t)\right) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1\left(s, v\left(s\right)\right) ds \end{pmatrix} \right\| \\
&= \left\| \begin{pmatrix} g_2\left(t, x(t)\right) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2\left(s, x\left(s\right)\right) ds - g_2\left(t, u(t)\right) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2\left(s, x\left(s\right)\right) ds \\ +g_2\left(t, u(t)\right) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2\left(s, x\left(s\right)\right) ds - g_2\left(t, u(t)\right) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2\left(s, u\left(s\right)\right) ds \\ \\ g_1\left(t, y(t)\right) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1\left(s, y\left(s\right)\right) ds - g_1\left(t, v(t)\right) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1\left(s, y\left(s\right)\right) ds \\ +g_1\left(t, v(t)\right) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1\left(s, y\left(s\right)\right) ds - g_1\left(t, v(t)\right) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1\left(s, v\left(s\right)\right) ds \end{pmatrix} \right\| \\
&= \left\| \begin{pmatrix} \left[g_2\left(t, x(t)\right) - g_2\left(t, u(t)\right)\right] \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} f_2\left(s, x\left(s\right)\right) ds \\ +g_2\left(t, u(t)\right) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \left[f_2\left(s, x\left(s\right)\right) - f_2\left(s, u\left(s\right)\right)\right] ds \\ \\ \left[g_1\left(t, y(t)\right) - g_1\left(t, v(t)\right)\right] \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f_1\left(s, y\left(s\right)\right) ds \\ +g_1\left(t, v(t)\right) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \left[f_1\left(s, y\left(s\right)\right) - f_1\left(s, v\left(s\right)\right)\right] ds \end{pmatrix} \right\| \\
&= \begin{pmatrix} \frac{L_2}{\Gamma(\beta)} \max_{t \in I} |x(t) - u(t)| \int_0^t (t-s)^{\beta-1} \left|f_2\left(s, x\left(s\right)\right)\right| ds \\ + \frac{K_2}{\Gamma(\beta)} \max_{t \in I} \left|g_2\left(t, u(t)\right)\right| |x(t) - u(t)| \int_0^t (t-s)^{\beta-1} ds \\ \\ \frac{L_1}{\Gamma(\alpha)} \max_{t \in I} |y(t) - v(t)| \int_0^t (t-s)^{\alpha-1} \left|f_1\left(s, y\left(s\right)\right)\right| ds \\ + \frac{K_1}{\Gamma(\alpha)} \max_{t \in I} \left|g_1\left(t, v(t)\right)\right| |y(t) - v(t)| \int_0^t (t-s)^{\alpha-1} ds \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{\beta\Gamma(\beta)} \left[L_2 N_2 + K_2 M_2\right] \|x(t) - u(t)\| \\ \\ \frac{1}{\alpha\Gamma(\alpha)} \left[L_1 N_1 + K_1 M_1\right] \|y(t) - v(t)\| \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{\Gamma(\beta+1)} \left[L_2 N_2 + K_2 M_2\right] \|x(t) - u(t)\| \\ \\ \frac{1}{\Gamma(\alpha+1)} \left[L_1 N_1 + K_1 M_1\right] \|y(t) - v(t)\| \end{pmatrix}
\end{aligned}$$

$$= \begin{pmatrix} r_1 \|x(t) - u(t)\| \\ \\ r_2 \|y(t) - v(t)\| \end{pmatrix}$$

where

$$r_1 = \frac{1}{\Gamma(\beta+1)} [L_2 N_2 + K_2 M_2], r_2 = \frac{1}{\Gamma(\alpha+1)} [L_1 N_1 + K_1 M_1]$$

which implies that

$$\|FX - FU\| \leq R \|X - U\|$$

where,

$$R = \max\{r_1, r_2\},$$

under the condition $0 < R < 1$, the mapping F is contraction and hence there exists a unique solution $X \in C^2(I)$ of the system (1.1) and this completes the proof. □

4.2. **Proof of convergence.**

Theorem 4.2. *Let the solution of the system (1.1) be exist. If $|x_{j1}(t)| < c$ where c is a positive constant then the series solution (3.3) of the system (1.1) using ADM converge.*

Proof. Define the two sequences $\{S_{1p}\}$ and $\{S_{2p}\}$ such that, $S_{1p} = \sum_{i=0}^{p} x_i(t)$ and $S_{2p} = \sum_{i=0}^{p} y_i(t)$ are the sequences of partial sums from the series solutions $\sum_{i=0}^{\infty} x_i(t)$ and $\sum_{i=0}^{\infty} y_i(t)$. Now,

$$\begin{aligned} g_1(t, y(t)) &= \sum_{i=0}^{\infty} A_i,\ f_1(s, y(s)) = \sum_{i=0}^{\infty} B_i, \\ g_2(t, x(t)) &= \sum_{i=0}^{\infty} C_i,\ f_2(s, x(s)) = \sum_{i=0}^{\infty} D_i, \end{aligned}$$

Let S_{jp} and S_{jq} ($j = 1, 2$), be arbitrary partial sums with $p > q$. We are going to prove that $\{S_{jp}\}$ are Cauchy sequences in this Banach space E.

$$\begin{aligned} \|S_{jp} - S_{jq}\| &= \left\| \begin{pmatrix} \sum_{i=0}^{p} x_i - \sum_{i=0}^{q} x_i \\ \\ \sum_{i=0}^{p} y_i - \sum_{i=0}^{q} y_i \end{pmatrix} \right\| \\ &\leq \|D\| \left\| \begin{pmatrix} \sum_{i=0}^{p} C_{i-1}(t) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \sum_{i=0}^{p} D_{i-1}(s)\,ds - \sum_{i=0}^{q} C_{i-1}(t) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \sum_{i=0}^{q} D_{i-1}(s)\,ds \\ \sum_{i=0}^{p} A_{i-1}(t) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \sum_{i=0}^{p} B_{i-1}(s)\,ds - \sum_{i=0}^{q} A_{i-1}(t) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \sum_{i=0}^{q} B_{i-1}(s)\,ds \end{pmatrix} \right\| \end{aligned}$$

$$\leq \left\| \begin{pmatrix} \sum\limits_{i=0}^{p} C_{i-1}(t) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \sum\limits_{i=0}^{p} D_{i-1}(s)\, ds - \sum\limits_{i=0}^{q} C_{i-1}(t) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \sum\limits_{i=0}^{p} D_{i-1}(s)\, ds \\ + \sum\limits_{i=0}^{q} C_{i-1}(t) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \sum\limits_{i=0}^{p} D_{i-1}(s)\, ds - \sum\limits_{i=0}^{q} C_{i-1}(t) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \sum\limits_{i=0}^{q} D_{i-1}(s)\, ds \\ \\ \sum\limits_{i=0}^{p} A_{i-1}(t) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \sum\limits_{i=0}^{p} B_{i-1}(s)\, ds - \sum\limits_{i=0}^{q} A_{i-1}(t) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \sum\limits_{i=0}^{p} B_{i-1}(s)\, ds \\ + \sum\limits_{i=0}^{q} A_{i-1}(t) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \sum\limits_{i=0}^{p} B_{i-1}(s)\, ds - \sum\limits_{i=0}^{q} A_{i-1}(t) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \sum\limits_{i=0}^{q} B_{i-1}(s)\, ds \end{pmatrix} \right\|$$

$$\leq \left\| \begin{pmatrix} \left[\sum\limits_{i=0}^{p} C_{i-1}(t) - \sum\limits_{i=0}^{q} C_{i-1}(t)\right] \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \sum\limits_{i=0}^{p} D_{i-1}(s)\, ds \\ + \sum\limits_{i=0}^{q} C_{i-1}(t) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \left[\sum\limits_{i=0}^{p} D_{i-1}(s) - \sum\limits_{i=0}^{q} D_{i-1}(s)\right] ds \\ \\ \left[\sum\limits_{i=0}^{p} A_{i-1}(t) - \sum\limits_{i=0}^{q} A_{i-1}(t)\right] \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \sum\limits_{i=0}^{p} B_{i-1}(s)\, ds \\ + \sum\limits_{i=0}^{q} A_{i-1}(t) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \left[\sum\limits_{i=0}^{p} B_{i-1}(s) - \sum\limits_{i=0}^{q} B_{i-1}(s)\right] ds \end{pmatrix} \right\|$$

$$\leq \left\| \begin{pmatrix} \left[\sum\limits_{i=q+1}^{p} C_{i-1}(t)\right] \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \sum\limits_{i=0}^{p} D_{i-1}(s)\, ds \\ + \sum\limits_{i=0}^{q} C_{i-1}(t) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \left[\sum\limits_{i=q+1}^{p} D_{i-1}(s)\right] ds \\ \\ \left[\sum\limits_{i=q+1}^{p} A_{i-1}(t)\right] \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \sum\limits_{i=0}^{p} B_{i-1}(s)\, ds \\ + \sum\limits_{i=0}^{q} A_{i-1}(t) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \left[\sum\limits_{i=q+1}^{p} B_{i-1}(s)\right] ds \end{pmatrix} \right\|$$

$$\leq \left\| \begin{pmatrix} \left[\sum\limits_{i=q}^{p-1} C_{i-1}(t)\right] \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \sum\limits_{i=0}^{p} D_{i-1}(s)\, ds \\ + \sum\limits_{i=0}^{q} C_{i-1}(t) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \left[\sum\limits_{i=q}^{p-1} D_{i-1}(s)\right] ds \\ \\ \left[\sum\limits_{i=q}^{p-1} A_{i-1}(t)\right] \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \sum\limits_{i=0}^{p} B_{i-1}(s)\, ds \\ + \sum\limits_{i=0}^{q} A_{i-1}(t) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \left[\sum\limits_{i=q}^{p-1} B_{i-1}(s)\right] ds \end{pmatrix} \right\|$$

$$\leq \left\| \begin{pmatrix} \left[g_2(t,S_{1(p-1)}) - g_2(t,S_{1(q-1)})\right] \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \left[f_2(t,S_{1p})\right] ds \\ +g_2(t,S_{1q}) \int_0^t \frac{(t-s)^{\beta-1}}{\Gamma(\beta)} \left[f_2(t,S_{1(p-)1}) - f_2(t,S_{1(q-1)})\right] ds \\ \\ \left[g_1(t,S_{2(p-1)}) - g_1(t,S_{2(q-1)})\right] \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \left[f_1(t,S_{2p})\right] ds \\ +g_1(t,S_{2q}) \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \left[f_1(t,S_{2(p-1)}) - f_1(t,S_{2(q-1)})\right] ds \end{pmatrix} \right\|$$

$$\leq \begin{pmatrix} \frac{1}{\Gamma(\beta+1)} \left[L_2N_2 + M_2K_2\right] \left\|S_{1(p-1)} - S_{1(q-1)}\right\| \\ \\ \frac{1}{\Gamma(\alpha+1)} \left[L_1N_1 + M_1K_1\right] \left\|S_{2(p-1)} - S_{2(q-1)}\right\| \end{pmatrix}$$

$$\leq R\left\|S_{j(p-1)} - S_{j(q-1)}\right\|$$

Let $p = q + 1$ then,

$$\left\|S_{j(q+1)} - S_{jq}\right\| \leq R\left\|S_{jq} - S_{j(q-1)}\right\| \leq R^2\left\|S_{j(q-1)} - S_{j(q-2)}\right\| \leq \cdots \leq R^q \left\|S_{j1} - S_{j0}\right\|$$

From the triangle inequality we have,

$$\begin{aligned} \|S_{jp} - S_{jq}\| &\leq \left\|S_{j(q+1)} - S_{jq}\right\| + \left\|S_{j(q+2)} - S_{j(q+1)}\right\| + \cdots + \left\|S_{jp} - S_{j(p-1)}\right\| \\ &\leq \left[R^q + R^{q+1} + \cdots + R^{p-1}\right] \|S_{j1} - S_{j0}\| \\ &\leq R^q\left[1 + R + \cdots + R^{p-q-1}\right] \|S_{j1} - S_{j0}\| \\ &\leq R^q\left[\frac{1 - R^{p-q}}{1-R}\right] \|x_{j1}\| \end{aligned}$$

where $\begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$. Now $0 < R < 1$, and $p > q$ implies that $(1 - R^{p-q}) \leq 1$. Consequently,

$$\begin{aligned} \|S_{jp} - S_{jq}\| &\leq \frac{R^q}{1-R} \|x_{j1}\| \\ &\leq \frac{R^q}{1-R} \max_{t\in I} |x_{j1}(t)| \end{aligned}$$

but, if $|x_{j1}(t)| < c$ then $\|S_{jp} - S_{jq}\| \to 0$ as $q \to \infty$ and hence, $\{S_{jp}\}$ are Cauchy sequences in this Banach space so, the series $\sum_{i=0}^{\infty} x_i(t)$ and $\sum_{i=0}^{\infty} y_i(t)$ converge and this completes the proof. □

5. NUMERICAL EXAMPLES

Example 1 Consider the following nonlinear FCSQIEs,

$$x(t) = \left(t^2 - \frac{t^{11/2}}{35\sqrt{\pi}}\right) + y^2(t) \int_0^t \frac{(t-s)^{-1/2}}{\Gamma(1/2)} y^3(s)\, ds, \tag{5.1}$$

$$y(t) = \left(\frac{t}{2} - \frac{1048576t^{39/2}}{22309287\sqrt{\pi}}\right) + x^4(t)\int_0^t \frac{(t-s)^{1/2}}{\Gamma(3/2)} x^5(s)\, ds,$$

and has the exact solution $x(t) = t^2, y(t) = \frac{t}{2}$.

Applying ADM to system (5.1), we get

$$x_0(t) = \left(t^2 - \frac{t^{11/2}}{35\sqrt{\pi}}\right),\ x_i(t) = A_{i-1}(t)\int_0^t \frac{(t-s)^{-1/2}}{\Gamma(1/2)} B_{i-1}(s)\, ds,\ i \geq 1,$$

$$y_0(t) = \left(\frac{t}{2} - \frac{1048576t^{39/2}}{22309287\sqrt{\pi}}\right),\ y_i(t) = C_{i-1}(t)\int_0^t \frac{(t-s)^{1/2}}{\Gamma(3/2)} D_{i-1}(s)\, ds,\ i \geq 1,$$

where A_i, B_i, C_i, and D_i are Adomian polynomials of the nonlinear terms y^2, y^3, x^4 and x^5 respectively and the solution will be,

$$x(t) = \sum_{i=0}^{q} x_i(t),\ y(t) = \sum_{i=0}^{q} y_i(t)$$

Table 1 shows the absolute error of ADM solution $(q = 2)$, while table 2 shows the absolute error of Picard solution $(q = 2)$.

Table 1: Absolute Error Table 2: Absolute Error

t	$\lvert x_{exact} - x_{ADM}\rvert$	$\lvert y_{exact} - y_{ADM}\rvert$
0.1	6.61744×10^{-24}	3.13306×10^{-26}
0.2	4.65868×10^{-20}	2.62749×10^{-19}
0.3	7.80598×10^{-16}	2.94604×10^{-15}
0.4	7.77967×10^{-13}	2.19741×10^{-12}
0.5	1.64741×10^{-10}	3.70791×10^{-10}
0.6	1.30963×10^{-8}	2.44052×10^{-8}
0.7	5.29431×10^{-7}	8.37533×10^{-7}
0.8	0.0000130294	0.0000178146
0.9	0.000217161	0.000262602
1	0.00249546	0.00290512

t	$\lvert x_{exact} - x_{Picard}\rvert$	$\lvert y_{exact} - y_{Picard}\rvert$
0.1	6.61744×10^{-24}	3.76158×10^{-37}
0.2	0	3.9443×10^{-31}
0.3	1.31798×10^{-18}	7.37112×10^{-26}
0.4	3.61304×10^{-15}	1.11707×10^{-20}
0.5	1.66655×10^{-12}	1.17449×10^{-16}
0.6	2.49761×10^{-10}	2.26918×10^{-13}
0.7	1.72135×10^{-8}	1.36183×10^{-10}
0.8	6.71089×10^{-7}	3.47021×10^{-8}
0.9	0.0000169013	4.56449×10^{-6}
1	0.000299432	0.000340497

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