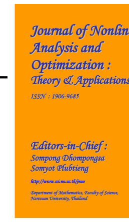

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COMMON FIXED POINT THEOREM IN INTUITIONISTIC FUZZY METRIC SPACE UNDER $(S-B)$ PROPERTY

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ABSTRACT. The main purpose of this paper is to give common fixed point theorem in intuitionistic fuzzy metric space under strict contractive conditions for mappings satisfying $(S-B)$ property.

KEYWORDS : Intuitionistic fuzzy metric space; Common fixed point; Compatible maps; Weakly compatible maps; $(S-B)$ property.
AMS Subject Classification: 47H10 54H25

1. INTRODUCTION

The concept of fuzzy sets was introduced initially by Zadeh [37] in 1965. Since, then to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. Atanassov [5] Introduced and studied the concept of intuitionistic fuzzy sets. Intuitionistic fuzzy sets as a generalization of fuzzy sets can be useful in situations when description of a problem by a (fuzzy) linguistic variable, given in terms of a membership function only, seems too rough. Coker [7] introduced the concept of intuitionistic fuzzy topological spaces. Alaca et al. [3] proved the well- known fixed point theorems of Banach [6] in the setting of intuitionisitc fuzzy metric spaces. Later on, Turkoglu et al. [35] Proved Jungcks [12] common fixed point theorem in the setting of intuitionisitc fuzzy metric space. Turkoglu et al. [35] further formulated the notions of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of pants theorem [20]. Gregori et al. [10], Saadati and Park [26] studied the concept of intuitionistic fuzzy metric space and its applications. Recently, many authors have also studied the fixed point theory in fuzzy and intuitionistic fuzzy metric space (See [9, 11, 23, 24, 31, 32, 33, 34, 36].

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The study of common fixed points of non compatible mappings is also very interesting. Work along these lines has recently been initiated by pant [21, 22]. Sharma and Bamboria [30] defined a property $(S-B)$ for self maps and obtained some common fixed point theorems for such mappings under strict contractive conditions. The class of $(S-B)$ maps contains the class of non compatible maps. Kamran [15] obtained some coincidence and fixed point theorems for hybrid strict contractions.

Definition 1.1. [27] A binary operation $* : [0,1] \times [0,1] \longrightarrow [0,1]$ is continuous t-norm if $*$ is satisfying the following condition;
(i) * is commutative and associative;
(ii) * is continuous
(iii) $a * 1 = a$ for all $a \in [0,1]$;
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 1.2. [27] A binary operation $\diamond : [0,1] \times [0,1] \longrightarrow [0,1]$ is continuous t-conorm if $\diamond$ is satisfying the following conditions;
(i) $\diamond$ is commutative and associative;
(ii) $\diamond$ is continuous
(iii) $a \diamond 0 = a$ for all $a \in [0,1]$;
(iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 1.3. A 5 tuple$(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, $\diamond$ is a continuous t-conorm X^2 and $M, N,$ are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions;
(i) $M(x,y,t) + N(x,y,t) \leq 1$ for all $x, y \leq X$ and $t > 0$;
(ii) $M(x,y,0) = 0$ for all $x, y \in X$;
(iii) $M(x,y,t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x,y,t) = M(y,x,t)$ for all $x, y \in X$ and $t > 0$;
(v) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$ for all $x, y, z \in X$ and $s, t > 0$;
(vi) for all $x, y \in X, M(x,y,\cdot)$ is left continuous;
(vii) $\lim_{t \longrightarrow \infty}$ for all $x, y \in X$ and $t > 0$;
(viii) $N(x,y,0) = 1$ for all $x, y \in X$;
(ix) $N(x,y,t) = 0$ for all $x, y \in X$ and $t > 0$ iff $x = y$;
(x) $N(x,y,t) = N(y,x,t)$ for all $x, y \in X$ and $t > 0$;
(xi) $N(x,y,t) \diamond N(y,z,s) \geq N(x,z,t+s)$ for all $x, y, z \in X$ and $s, t > 0$;
(xii) for all $x, y \in X, N(x,y,\cdot) : [0,\infty) \longrightarrow [0,1]$ is right continuous;
(xiii) $\lim_{t \longrightarrow \infty} N(x,y,t) = 0$ for all x, y in X.
(M, N) is called an intuitionistic fuzzy metric on X.
The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of nonnearness between x and y with respect to t respectively.

Remark 1.4. An intuitionistic fuzzy metric spaces with continous t-norm * and continous t-conorm $\diamond$ defined by $a * a \geq a$ and $(1-a) \diamond (1-a) \leq (1-a)$ for all $a \in [0,1]$. Then for all $x, y \in X, M(x,y,*)$ is non- decreasing and $N(x,y,\diamond)$ is nonincreasing.

Alaca, Turkoglu and Yildiz [3] introduced the following notions;

Definition 1.5. Let $(X, M, N*, \diamond)$ be an intuitionistic fuzzy metric space. Then (a) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$, $lim_{n \longrightarrow \infty} M(x_{n+p}, x_n, t) = 1, lim_{n \longrightarrow \infty} N(x_{n+p}, x_n, t) = 0$. (b) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if , for all $t > 0$, $lim_{n \longrightarrow \infty} M(x_n, x, t) = 1, lim_{n \longrightarrow \infty} N(x_n, x, t) = 0$;

Since * and $\diamond$ are continuous, the limit is uniquely determined from (v) and (xi) of definition 1.3, respectively. An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 1.6. [32] A pair of self mappings (f, g) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be compatible if $\lim_{n\longrightarrow\infty} M(fgx_n, gfx_n, t) = 1$ and $\lim_{n\longrightarrow\infty} N(fgx_n, gfx_n, t) = 0$ for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\longrightarrow\infty} fx_n = \lim_{n\longrightarrow\infty} gx_n = z$ for some $z \in X$.

Definition 1.7. A pair of self mappings (f, g) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be non-compatible $\lim_{n\longrightarrow\infty} M(fgx_n, gfx_n, t) \neq 1$ or non existent and $\lim_{n\longrightarrow\infty} N(fgx_n, gfx_n, t) \neq 0$ or non existent for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\longrightarrow\infty} fx_n = \lim_{n\longrightarrow\infty} gx_n = z$ for some $z \in X$.

In 1998, Jungck and Rhodes [14] introduced the concept of weakly compatible maps as follows;

Definition 1.8. Two self maps f and g are said to be weakly compatible if they commute at coincidence points.

Aamri and Moutawakil [1] generalized the concept of non compatibility in metric spaces by defining the notion of E.A. Property and proved common fixed point theorems using this property.

Sharma and Bamboria [30] defined the $(S - B)$ property and proved common fixed point theorems in fuzzy metric spaces using this property.

Definition 1.9. A pair of self mappings (S, T) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to satisfy $(S - B)$ property if there exists a sequence $\{x_n\}$ is X such that

$$\lim_{n\longrightarrow\infty} M(Sx_n, Tx_n, t) = 1, \lim_{n\longrightarrow\infty} N(Sx_n, Tx_n, t) = 0$$

Example 1.10. Let $X = [0, \infty)$ consider $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space, where M and N are two fuzzy sets defined by $M(x, y, t) = t/[t + d(x, y)]$ and $N(x, y, t) = d(x, y)/[t + d(x, y)]$ where d is usual metric. Define $T, S : X \longrightarrow [0, \infty)$ by $Tx = x/5$ and $Sx = 2x/5$ for all x in X. Consider $x_n = 1/n$. Now, $\lim_{n\longrightarrow\infty} M(Sxn, Txn, t) = 1$ and $\lim_{n\longrightarrow\infty} N(Sxn, Txn, t) = 0$. Therefore S and T satisfy property $(S - B)$.

Lemma 1.11. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and for all $x, y \in X, t > 0$ and if for a number $k \in (0, 1)$, $M(x, y, kt).M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$ then $x = y$.*

Lemma 1.12. [2] *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exist is a number $k \in (0, 1)$ such that;*

$$M(y_{n+2}, y_{n+1}, kt).M(y_{n+1}, y_n, t)$$

and

$$N(y_{n+2}, y_{n+1}, kt).N(y_{n+1}, y_n, t)$$

for all $t > 0$ and $n = 1, 2, \ldots$ Then $\{y_n\}$ is a Cauchy sequence in X.

2. MAIN RESULTS

Theorem 2.1. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space with $a*a \geq a$ and $(1-a) \diamond (1-a) \leq (1-a)$ for all $a \in [0,1)$, let A, B, S, and T be self mappings of X into itself such that,*

(1.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$,

(1.2) (A, S) or (B, T) satisfies the property $(S-B)$.

(1.3) there exists a number $k \in (0,1)$ such that

$$[1 + PM(Sx, Ty, kt)] * M(Ax, By, kt)$$
$$\geq \acute{\phi}[PM(Ax, Sx, kt) * M(By, Ty, kt) + M(Ax, Ty, kt) * M(By, Sx, kt) + M(Sx, Ty, t)]$$
$$*M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, t) * M(Ax, Ty, (2-\alpha)t)$$

and

$$[1 + PN(Sx, Ty, kt)] \diamond N(Ax, By, kt)$$
$$\leq \psi[PN(Ax, Sx, kt) \diamond N(By, Ty, kt) + N(Ax, Ty, kt) \diamond N(By, Sx, kt) + N(Sx, Ty, t)$$
$$\diamond(N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(By, Sx, t) \diamond N(Ax, Ty, (2-a)t)]$$

for all $x, y \in X, P \geq 0$, $\alpha \in (0,2)$ and $t > 0$. Where $p\acute{h}i, \psi : [0,1] \longrightarrow [0,1]$ is continuous function such that $\acute{\phi}(S) > S$ and $\psi(S) < S$ for each $0 < S < 1$ with $M(x, y, t) > 0$.

(1.4) the pairs (A, S) and (B, T) are weakly compatible,

(1.5) one of A(X), B(X), S(X) or T(X) is a closed subset of X. Then A, B, S and T have a unique common fixed point in X.

Proof *Suppose that (B, T) satisfies the $(S-B)$ property, then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Tx_n = z$ for some $z \in X$. Since $B(X) \subset S(X)$ there exists a sequence $\{y_n\} \in X$ such that $\lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Sy_n = z$. Now we shall show that $\lim_{n\longrightarrow\infty} Ay_n = z$. From (1.3) for $f\alpha = 1-q, q \in (0,1)$ we have;*

$$[1 + PM(Sy_n, Tx_n, kt)] * M(Ay_n, Bx_n, kt)$$
$$\acute{\phi}[PM(Ay_n, Sy_n, kt) * M(Bx_n, Tx_n, kt) + M(Ay_n, Tx_n, kt) * M(Bx_n, Sy_n, kt)$$
$$+M(Sy_n, Tx_n, t) * M(Ay_n, Sy_n, t) * M(Bx_n, Tx_n, t) * M(Bx_n, Sy_n, t)$$
$$*M(Ay_n, Tx_n(2-\alpha)t)]$$

and

$$[1 + PN(Sy_n, Tx_n, kt)] * N(Ay_n, Bx_n, kt)$$
$$\leq \psi[PN(Ay_n, Sy_n, kt) \diamond N(Bx_n, Tx_n, kt) + N(Ay_n, Tx_n, kt) \diamond N(Bx_n, Sy_n, kt)$$
$$+N(Sy_n, Tx_n, t) \diamond N(Ay_n, Sy_n, t) \diamond N(Bx_n, Tx_n, t) \diamond N(Bx_n, Sy_n, t)$$
$$\diamond N(Ay_n, Tx_n, (2-\alpha)t)]$$

$$M(Ay_n, Bx_n, kt) + P[M(Sy_n, Tx_n, kt) * M(Ay_n, Bx_n, kt)]$$
$$\geq \acute{\phi}[PM(Ay_n, Sy_n, kt) * M(Bx_n, Tx_n, kt) + M(Ay_n, Tx_n, kt) * M(Bx_n, Sy_n, kt)$$
$$+M(Sy_n, Tx_n, t) * M(Ay_n, Sy_n, t) * M(Bx_n, Tx_n, t)$$
$$*M(Bx_n, Sy_n, t) * M(Ay_n, Tx_n, (1+q)t)]$$

and

$$N(Ay_n, Bx_n, kt) + P[N(Sy_n, Tx_n, kt) \diamond N(Ay_n, Bx_n, kt)]$$
$$\leq \psi[PN(Ay_n, Sy_n, kt) \diamond N(Bx_n, Tx_n, kt) + N(Ay_n, Tx_n, kt) \diamond N(Bx+n, Sy_n, kt)$$
$$+N(Sy_n, Tx_n, t) \diamond N(Ay_n, Sy_n, t)$$

$$\diamond N(Bx_n, Tx_n, t) \diamond N(Bx_n, Sy_n, t) \diamond N(Ay_n, Tx_n, (1+q)t)]$$

$$M(Ay_n, Bx_n, kt) + P[M(Bx_n, Tx_n, kt) * M(Ay_n, Bx_n, kt)]$$
$$\geq \phi[PM(Ay_n, Bx_n, kt) * M(Bx_n, Tx_n, kt) + M(Ay_n, Tx_n, kt) * M(Bx_n, Bx_n, kt)$$
$$+M(Bx_n, Tx_n, t) * M(Ay_n, Bx_n, t) * M(Bx_n, Tx_n, t)$$
$$*M(Bx_n, Bx_n, t) * M(Ay_n, Bx_n, t) * M(Bx_n, Tx_n, qt)]$$

and

$$N(Ay)n, Bx_n, kt) + P[N(Bx_n, Tx_n, kt) \diamond N(Ay_n, Bx_n, kt)]$$
$$\leq \psi[PN(Ay_n, Bx_n, kt) \diamond N(Bx_n, Tx_n, kt) + N(Ay_n, Tx_n, kt) \diamond N(Bx_n, Bx_n, kt)$$
$$+N(Bx_n, Tx_n, t) \diamond N(Ay_n, Bx_n, t) \diamond N(Bx_n, Tx_n, t) \diamond N(Bx_n, Bx_n, t)$$
$$\diamond N(Ay_n, Bx_n, t) \diamond N(Bx_n, Tx_n, qt)]$$

Thus it follows that,

$$M(Ay_n, Bx_n, kt) \geq \phi[M(Bx_n, Tx_n, t) * M(Ay_n, Bx_n, t) * M(Bx_n, Tx_n, qt)]$$

and

$$N(Ay_n, Bx_n, kt) \leq \psi[N(Bx_n, Tx_n, t) \diamond N(Ay_n, Bx_n, t)) \diamond N(Bx_n, Tx_n, qt)]$$

*Since the t-norm * and t-conorm $\diamond$ is continuous and $M(x, y, \cdot)$ and $N(x, y, \cdot)$ is continuous, letting $q \longrightarrow 1$ we have,*

$$M(Ay_n, Bx_n, kt) \geq \phi[M(Bx_n, Tx_n, t) * M(Ay_n, Bx_n, t)]$$

and

$$N(Ay_n, Bx_n, kt) \leq \psi[N(Bx_n, Tx_n, t) \diamond N(Ay_n, Bx_n, t)]$$

It follows that

$$\begin{aligned} \lim_{n\longrightarrow\infty} M(Ay_n, Bx_n, kt) &\geq \phi[\lim_{n\longrightarrow\infty} M(Ay_n, Bx_n, t), M(\lim_{n\longrightarrow\infty} Ay_n, z, kt)] \\ &> M(\lim_{n\longrightarrow\infty} Ay_n, z, t) \end{aligned}$$

and

$$\begin{aligned} \lim_{n\longrightarrow\infty} N(Ay_n, Bx_n, kt) &\leq \psi[\lim_{n\longrightarrow\infty} N(Ay_n, Bx_n, t), N(\lim_{n\longrightarrow\infty} Ay_n, z, kt)] \\ &< N(\lim_{n\longrightarrow\infty} Ay_n, z, t) \end{aligned}$$

and we deduce that $\lim_{n\longrightarrow\infty} Ay_n = z$. Suppose $S(X)$ is a closed subset of X. Then $z = Su$ for some $u \in X$. Subsequently we have,

$$\lim_{n\longrightarrow\infty} Ay_n = \lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Tx_n = \lim_{n\longrightarrow\infty} Sy_n = Su.$$

By (1.3) with $f\alpha = 1$, we have;

$$[1 + PM(Su, Tx_n, kt)] * M(Au, Bx_n, kt)$$
$$\geq \phi[PM(Au, Su, kt) * M(Bx_n, Tx_n, kt) + M(Bx_n, Su, t) * M(Au, Tx_n, t)$$
$$+M(Su, Tx_n, t) * M(Au, Su, t) * M(Bx_n, Tx_n, t) * M(Bx_n, Su, t) * M(Au, Tx_n, t)]$$

and

$$[1 + PN(Su, Tx_n, kt)] \diamond N(Au, Bx_n, kt)$$
$$\leq \psi[PN(Au, Su, kt) \diamond N(Bx_n, Tx_n, kt) + N(Bx_n, Su, t) \diamond N(Au, Tx_n, t) +$$
$$N(Su, Tx_n, t) \diamond N(Au, Su, t) \diamond N(Bx_n, Tx_n, t) \diamond N(Bx_n, Su, t) \diamond N(Au, Tx_n, t)]$$

$$M(Au, Bx_n, kt) + P[M(Su, Tx_n, kt) * M(Au, Bx_n, kt)]$$
$$\geq \phi[P[M(Au, Su, kt) * M(Bx_n, Tx_n, kt) + M(Au, Tx_n, kt) * M(Bx_n, Su, kt)]$$

$$+M(Su,Tx_n,kt)*M(Au,Su,t)*M(Bx_n,Tx_n,t)*M(Bx_n,Su,t)*M(Au,Tx_n,t)]$$

and

$$N(Au,Bx_n,kt)+P[N(Su,Tx_n,kt))\diamond N(Au,Bx_n,kt)]$$
$$\leq \psi[PN(Au,Su,kt)\diamond N(Bx_n,Tx_n,kt)+N(Au,Tx_n,kt)\diamond N(Bx_n,Su,kt)$$
$$+[N(Su,Tx_n,kt)\diamond N(Au,Su,t)\diamond N(Bx_n,Tx_n,t)\diamond N(Bx_n,Su,t))\diamond N(Au,Tx_n,t)]$$

Taking the $\lim_{n\longrightarrow\infty}$ *we have;*

$$M(Au,Su,kt)\geq \phi[PM(Au,Su,kt)*M(Su,Su,kt)+M(Su,Su,t)*$$
$$M(Au,Su,t)*M(Su,Su,t)*M(Su,Su,t)*M(Au,Su,t)]$$

and

$$N(Au,Su,kt) \leq \psi[PN(Au,Su,kt)\diamond N(Su,Su,kt)+N(Su,Su,t)\diamond$$
$$N(Au,Su,t)\Diamond N(Su,Su,t)\Diamond N(Su,Su,t)\Diamond N(Au,Su,t)]$$

This gives

$$M(Au,Su,kt)>M(Au,Su,t)$$

and

$$N(Au,Su,kt)<N(Au,Su,t)$$

Therefore by lemma 1, we have $Au=Su$. *I.e.* A *and* S *have a coincidence point. The weak compatibility of* A *and* S *implies that* $ASu=SAu$ *and then*

$$AAu=ASu=SAu=SSu.$$

On the other hand, since $A(X)\subset T(X)Y$, *there exists a point* $v\in X$ *such that* $Au=Tv$. *We claim that* $Tv=Bv$ *using (1.3) with* $\alpha=1$, *we have,*

$$[1+PM(Su,Tv,kt)]*M(Au,Bv,kt)$$
$$\geq \acute{\phi}[PM(Au,Su,kt)*M(Bv,Tv,kt)+M(Au,Tv,kt)*M(Bv,Su,kt)+$$
$$M(Su,Tv,t)*M(Au,Su,t)*M(Bv,Tv,t)*M(Bv,Su,t)*M(Au,Tv,t)]$$

and

$$[1+PN(Su,Tv,kt)]\Diamond N(Au,Bv,kt)$$
$$\leq \Psi[PN(Au,Su,kt)\Diamond N(Bv,Tv,kt)+N(Au,Tv,kt)\Diamond N(Bv,Su,kt)+N(Su,Tv,t)\Diamond$$
$$N(Au,Su,t)\Diamond N(Bv,Tv,t)\Diamond N(Bv,Su,t)\Diamond N(Au,Tv,t)]$$

$$M(Au,Bv,kt)+P[M(Su,Tv,kt)*M(Au,Bv,kt)]$$
$$\geq \acute{\phi}[PM(Au,Su,kt)*M(Bv,Tv,kt)+M(Au,Tv,kt)*M(Bv,Su,kt)+$$
$$M(Su,Tv,t)*M(Au,Su,t)*M(Bv,Tv,t)*M(Bv,Su,t)*M(Au,Tv,t)]$$

And

$$N(Au,Bv,kt)+P[N(Su,Tv,kt)N(Au,Bv,kt)]$$
$$\leq \Psi[PN(Au,Su,kt)\Diamond N(Bv,Tv,kt)+N(Au,Tv,kt).N(Bv,Su,kt)+$$
$$N(Su,Tv,t).N(Au,Su,t)\Diamond N(Bv,Tv,t)\Diamond N(Bv,Su,t)\Diamond N(Au,Tv,t)]$$

Thus it follows that, $M(Au,Bv,kt)>M(Au,Bv,t)$ *and*

$$N(Au,Bv,kt)<N(Au,Bv,t)$$

$$M(AAu,Bv,kt)+P[M(SAu,Tv,kt)*M(AAu,Bv,kt)]$$
$$\geq \acute{\phi}[PM(AAu,SAu,kt)*M(Bv,Tv,kt)+M(AAu,Tv,kt)*M(Bv,SAu,kt)$$
$$+M(SAu,Tv,t)*M(AAu,SAu,t)*M(Bv,Tv,t)*M(Bv,SAu,t)*M(AAu,Tv,t)]$$

and

$$N(AAu, Bv, kt) + P[N(SAu, Tv, kt)\Diamond N(AAu, Bv, kt)]$$
$$\leq \Psi[PN(AAu, SAu, kt)\Diamond N(Bv, Tv, kt) + N(AAu, Tv, kt)\Diamond N(Bv, SAu, kt)$$
$$+N(SAu, Tv, t)\Diamond N(AAu, SAu, t)\Diamond N(Bv, Tv, t)\Diamond N(Bv, SAu, t)\Diamond N(AAu, Tv, t)]$$

$$M(AAu, Au, kt) + P[M(AAu, Au, kt) * M(AAu, Au, kt)]$$
$$\geq \acute{\phi}[PM(AAu, AAu, kt) * M(Au, Au, kt) + M(AAu, Au, kt) * M(Au, AAu, kt)$$
$$+M(AAu, Au, t) * M(AAu, AAu, t) * M(Au, Au, t)$$
$$*M(Au, Au, t) * M(AAu, Au, t)]$$

and

$$N(AAu, Au, kt) + P[N(AAu, Au, kt)\Diamond N(AAu, Au, kt)]$$
$$\leq \Psi[PN(AAu, AAu, kt)\Diamond N(Au, Au, kt) + N(AAu, Au, kt)\Diamond N(Au, AAu, kt)$$
$$+N(AAu, Au, t)\Diamond N(AAu, AAu, t)\Diamond N(Au, Au, t)$$
$$\Diamond N(Au, AAu, t)\Diamond N(AAu, Au, t)]$$

Thus it follows that

$$M(AAu, Au, kt) > M(AAu, Au, t)$$

and

$$N(AAu, Au, kt) < N(AAu, Au, t)$$

Therefore by lemma 1, we have $Au = AAu = SAu$. *I.e.* Au *is a common fixed point of* A *and* S. *Similarly, we prove that* Bv *is a common fixed point of* B *and* T. *Since* $Au = Bv$, *we conclude that* Au *is a common fixed point of* A, B, S *and* T. *If* $Au = Bu = Su = Tu = u$ *and* $Av = Bv = Sv = Tv = v$, *then by (1.3) with* $\alpha = 1$, *we have;*

$$[1 + PM(Su, Tv, kt)] * (Au, Bv, kt)$$
$$\geq \acute{\phi}[PM(Au, Su, kt) * M(Bv, Tv, kt) + M(Au, Tv, kt) * M(Bv, Su, kt) +$$
$$M(Su, Tv, t) * M(Au, Su, t) * M(Bv, Tv, t) * M(Bv, Su, t) * M(Au, Tv, t)]$$

$$[1 + PN(Su, Tv, kt)]\Diamond(Au, Bv, kt)$$
$$\leq \Psi[PN(Au, Su, kt)\Diamond N(Bv, Tv, kt) + N(Au, Tv, kt)\Diamond N(Bv, Su, kt) +$$
$$N(Su, Tv, t).N(Au, Su, t)\Diamond N(Bv, Tv, t)\Diamond N(Bv, Su, t)\Diamond N(Au, Tv, t)]$$

$$M(u, v, kt) + P[M(u, v, kt) * M(u, v, kt)]$$
$$\geq \acute{\phi}[PM(u, v, kt) * M(v, v, kt) + M(u, v, kt) * M(v, u, kt) + M(u, v, t) *$$
$$M(u, u, t) * M(v, v, t) * M(v, u, t) * M(u, v, t)]$$

and

$$N(u, v, kt) + P[N(u, v, kt).N(u, v, kt)]$$
$$\leq \Psi[PN(u, u, kt)\Diamond N(v, v, kt) + N(u, v, kt)\Diamond N(v, u, kt) + N(u, v, t)$$
$$\Diamond N(u, u, t)\Diamond N(v, v, t)\Diamond N(v, u, t)\Diamond N(u, v, t)]$$
$$M(u, v, kt) > M(u, v, t)$$

and

$$N(u, v, kt) < N(u, v, t)$$

By lemma 1, we have $u = v$. *Hence the common fixed point is a unique. This completes the proof of the theorem.*

Corollary 2.2. *Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space with $a*a \geq a$ and $(1-a)\diamondsuit(1-a) \leq (1-a)$ for all $a \in (0,1)$, let A, B, S, and T be self mappings of X into itself such that;*

(1.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$

(1.2) (A, S) or (B, T) satisfies the property $(S - B)$.

(1.3) there exists a number $k \in (0,1)$ such that

$$M(Ax, By, kt) \geq \acute{\phi}[M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, t) * M(Ax, Ty, (2-\alpha)t)]$$

and

$$N(Ax, By, kt) \leq \Psi[N(Sx, Ty, t)\diamondsuit N(Ax, Sx, t)\diamondsuit N(By, Ty, t)\diamondsuit N(By, Sx, t)\diamondsuit N(Ax, Ty, (2-\alpha)t)]$$

for all $x, y \in X$, $P \geq 0, \alpha \in (0,2)$ and $t > 0$. Where $\acute{\phi}, \Psi : [0,1] \to [0,1]$ is continuous function such that $\acute{\phi}(S) > S$ and $f\Psi(S) < S$ for each $0 < S < 1$ with $M(x, y, t) > 0$.

(1.4) the pairs $\{A, S\}$ and (B, T) are weakly compatible;

(1.5) one of $A(X), B(X), S(X)$ or $T(X)$ is a closed subset of X. Then A, B, S, and T have a unique common fixed point in X.

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