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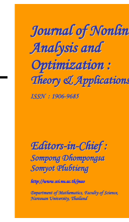

SOME COMMON FIXED POINT RESULTS FOR GENERALIZED WEAK CONTRACTIVE MAPPINGS IN PARTIALLY ORDERED METRIC SPACES

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ABSTRACT. Some common fixed point results satisfying a generalized weak contractive condition in the framework of partially ordered metric spaces are obtained. The proved results generalize and extend some known results in the literature.

KEYWORDS : Common fixed point; f-Weakly contractive mappings; (μ, ψ)-Generalized f-weakly contractive mappings; Weakly compatible mappings
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1. INTRODUCTION AND PRELIMINARIES

Fixed point theory is an old and rich branch of analysis and has a large number of applications. Fixed point problems involving different contractive type inequalities have been studied by many authors (see [1]-[20] and references cited therein). The main aim of this work is to prove some common fixed point theorems for (μ, ψ)-generalized f-weakly contractive mappings in partially ordered metric spaces.

The Banach contraction mapping is one of the pivotal results of analysis. It is very popular tool for solving existence problems in many different fields of mathematics. There are a lot of generalizations of the Banach contraction principle in the literature. Ran and Reurings [18] extended the Banach contraction principle in partially ordered sets with some applications to linear and nonlinear matrix equations. While Nieto and Rodŕiguez-López [17] extended the result of Ran and Reurings and applied their main theorems to obtain a unique solution for a first order ordinary differential equation with periodic boundary conditions. Bhaskar and Lakshmikantham [2] introduced the concept of mixed monotone mappings and obtained some coupled fixed point results. Also, they applied their results on a first order differential equation with periodic boundary conditions.

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Alber and Guerre-Delabriere [1] introduced the concept of weakly contractive mappings and proved the existence of fixed points for single-valued weakly contractive mappings in Hilbert spaces. Thereafter, in 2001, Rhoades [20] proved the fixed point theorem which is one of the generalizations of Banach's Contraction Mapping Principle, because the weakly contractions contains contractions as a special case and he also showed that some results of [1] are true for any Banach space. In fact, weakly contractive mappings are closely related to the mappings of Boyd and Wong [3] and of Reich types [19]. Fixed point problems involving weak contractions and mappings satisfying weak contractive type inequalities have been studied by many authors (see [1], [7]-[15], [20] and references cited therein).

First, we recall some basic definitions and related results.

A map $T : X \longrightarrow X$ is called a *weakly contractive mapping* (see [1], [13], [20]) if for each $x, y \in X$,

$$d(Tx, Ty) \leq d(x, y) - \psi(d(x, y)) \tag{1.1}$$

where $\psi : [0, \infty) \longrightarrow [0, \infty)$ is continuous and nondecreasing, $\psi(x) = 0$ if and only if $x = 0$ and $\lim \psi(x) = \infty$.

If we take $\psi(x) = kx$, $0 < k < 1$, then a weakly contractive mapping is called a contraction.

A map $T : X \longrightarrow X$ is called a *f-weakly contractive mapping* (see [14]) if for each $x, y \in X$,

$$d(Tx, Ty) \leq d(fx, fy) - \psi(d(fx, fy)) \tag{1.2}$$

where $f : X \longrightarrow X$ is a self-mapping, $\psi : [0, \infty) \longrightarrow [0, \infty)$ is continuous and nondecreasing, $\psi(x) = 0$ if and only if $x = 0$ and $\lim \psi(x) = \infty$.

If we take $\psi(x) = (1 - k)x$, $0 < k < 1$, then a f-weakly contractive mapping is called a f-contraction. Further, if $f =$ identity mapping and $\psi(x) = (1 - k)x$, $0 < k < 1$, then a f-weakly contractive mapping is called a contraction.

A map $T : X \longrightarrow X$ is called a *generalized f-weakly contractive mapping* (see [7]) if for each $x, y \in X$,

$$d(Tx, Ty) \leq \frac{1}{2}[d(fx, Ty) + d(fy, Tx)] - \psi(d(fx, Ty), d(fy, Tx)) \tag{1.3}$$

where $f : X \longrightarrow X$ is a self-mapping, $\psi : [0, \infty)^2 \longrightarrow [0, \infty)$ is a continuous mapping such that $\psi(x, y) = 0$ if and only if $x = y = 0$.

If $f =$identity mapping, then a generalized f-weakly contractive mapping is a generalized weakly contractive mapping (see [13]).

Khan et al. [16] initiated the use of a control function that alters distance between two points in a metric space, which they called an altering distance function.

A function $\mu : [0, \infty) \longrightarrow [0, \infty)$ is called an *altering distance function* if the following properties are satisfied:

(i) μ is monotone increasing and continuous;

(ii) $\mu(t) = 0$ if and only if $t = 0$.

A map $T : X \longrightarrow X$ is called a *(μ, ψ)-generalized f-weakly contractive mapping* (see [8]) if for each $x, y \in X$,

$$\mu(d(Tx, Ty)) \leq \mu(\frac{1}{2}[d(fx, Ty) + d(fy, Tx)]) - \psi(d(fx, Ty), d(fy, Tx)) \tag{1.4}$$

where $f : X \longrightarrow X$ is a self-mapping, $\mu : [0, \infty) \longrightarrow [0, \infty)$ is an altering distance function and $\psi : [0, \infty)^2 \longrightarrow [0, \infty)$ is a lower semi-continuous mapping such that $\psi(x, y) = 0$ if and only if $x = y = 0$.

If $f =$identity mapping, then a (μ, ψ)-generalized f-weakly contractive mapping is a *(μ, ψ)-generalized weakly contractive mapping.*

Let M be a nonempty subset of a metric space (X, d), a point $x \in M$ is a *common fixed (coincidence) point* of f and T if $x = fx = Tx$ $(fx = Tx)$. The set of fixed points (respectively, coincidence points) of f and T is denoted by $F(f, T)$ (respectively, $C(f, T)$). The mappings $T, f : M \to M$ are called *commuting* if $Tfx = fTx$ for all $x \in M$; *compatible* if $\lim d(Tfx_n, fTx_n) = 0$ whenever $\{x_n\}$ is a sequence such that $\lim Tx_n = \lim fx_n = t$ for some t in M; *weakly compatible* if they commute at their coincidence points, i.e., if $fTx = Tfx$ whenever $fx = Tx$.

Suppose $(X, \leq)$ is a partially ordered set and $T, f : X \longrightarrow X$. A mapping T is said to be *monotone f-nondecreasing* if for all $x, y \in X$,

$$fx \leq fy \text{ implies } Tx \leq Ty. \tag{1.5}$$

If f=identity mapping, then T is a *monotone nondecreasing*.

A subset W of a partially ordered set X is said to be *well ordered* if every two elements of W are comparable.

2. MAIN RESULTS

Theorem 2.1. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Suppose that T and f are self mappings on X, $T(X) \subseteq f(X)$, T is a monotone f-nondecreasing mapping and*

$$\mu(d(Tx, Ty)) \leq \mu(\frac{1}{2}[d(fx, Ty) + d(fy, Tx)]) - \psi(d(fx, Ty), d(fy, Tx)) \tag{2.1}$$

for all $x, y \in X$ for which $f(x) \geq f(y)$ where $\mu : [0, \infty) \longrightarrow [0, \infty)$ is an altering distance function and $\psi : [0, \infty)^2 \longrightarrow [0, \infty)$ is a lower semi-continuous mapping such that $\psi(x, y) = 0$ if and only if $x = y = 0$.

If $\{f(x_n)\} \subset X$ is a nondecreasing sequence with $f(x_n) \longrightarrow f(z)$ in $f(X)$, then $f(x_n) \leq f(z)$, and $f(z) \leq f(f(z))$ for every n.

Also suppose that $f(X)$ is closed. If there exists an $x_0 \in X$ with $f(x_0) \leq T(x_0)$, then T and f have a coincidence point.

Further, if T and f are weakly compatible, then T and f have a common fixed point. Moreover, the set of common fixed points of T and f is well ordered if and only if T and f have one and only one common fixed point.

Proof. Let $x_0 \in X$ such that $f(x_0) \leq T(x_0)$. Since $T(X) \subseteq f(X)$, we can choose $x_1 \in X$ so that $fx_1 = Tx_0$. Since $Tx_1 \in f(X)$, there exists $x_2 \in X$ such that $fx_2 = Tx_1$. By induction, we construct a sequence $\{x_n\}$ in X such that $fx_{n+1} = Tx_n$, for every $n \geq 0$.

Since $f(x_0) \leq T(x_0)$, $T(x_0) = f(x_1)$, $f(x_0) \leq f(x_1)$, T is monotone f-nondecreasing mapping, $T(x_0) \leq T(x_1)$. Similarly $f(x_1) \leq f(x_2)$, $T(x_1) \leq T(x_2)$, $f(x_2) \leq f(x_3)$. Continuing, we obtain

$$T(x_0) \leq T(x_1) \leq T(x_2) \leq \ldots \leq T(x_n) \leq T(x_{n+1}) \leq \ldots.$$

We suppose that $d(T(x_n), T(x_{n+1})) > 0$ for all n. If not then $T(x_{n+1}) = T(x_n)$ for some n, $T(x_{n+1}) = f(x_{n+1})$, i.e. T and f have a coincidence point x_{n+1}, and so we have the result.

Consider

$$\begin{aligned}\mu(d(Tx_{n+1}, Tx_n)) &\leq \mu(\frac{1}{2}[d(fx_{n+1}, Tx_n) + d(fx_n, Tx_{n+1})]) - \psi(d(fx_{n+1}, Tx_n), d(fx_n, Tx_{n+1})) \\ &= \mu(\frac{1}{2}d(Tx_{n-1}, Tx_{n+1})) - \psi(0, d(Tx_{n-1}, Tx_{n+1})) \qquad (*) \\ &\leq \mu(\frac{1}{2}d(Tx_{n-1}, Tx_{n+1}))\end{aligned}$$

$$\leq \quad \mu(\frac{1}{2}[d(Tx_{n-1},Tx_n)+d(Tx_n,Tx_{n+1})])$$

Since μ is a non-decreasing function, for all $n = 1, 2 \ldots$, we have $d(Tx_{n+1}, Tx_n) \leq d(Tx_n, Tx_{n-1})$. Thus $\{d(Tx_{n+1}, Tx_n)\}$ is a monotone decreasing sequence of non-negative real numbers and hence is convergent. Hence there exists $r \geq 0$ such that $d(Tx_{n+1}, Tx_n) \longrightarrow r$.

From inequality $(*)$, we have

$$\begin{aligned} d(Tx_{n+1},Tx_n) &\leq \frac{1}{2}d(Tx_{n-1},Tx_{n+1}) \\ &\leq \frac{1}{2}[d(Tx_{n-1},Tx_n)+d(Tx_n,Tx_{n+1})] \end{aligned}$$

letting $n \longrightarrow \infty$, we have

$$r \leq \lim \frac{1}{2}d(Tx_{n-1},Tx_{n+1}) \leq \frac{1}{2}r+\frac{1}{2}r,$$

i.e. $\lim d(Tx_{n-1}, Tx_{n+1}) = 2r$. Using the continuity of μ and lower semi-continuity of ψ, and inequality $(*)$, we have $\mu(r) \leq \mu(r) - \psi(0, 2r)$, and consequently, $\psi(0, 2r) \leq 0$. Thus $r = 0$. Hence

$$d(Tx_{n+1},Tx_n) \longrightarrow 0.$$

Now, we show that $\{Tx_n\}$ is a Cauchy sequence. If otherwise, then there exists $\epsilon > 0$ for which we can find subsequences $\{Tx_{m(k)}\}$ and $\{Tx_{n(k)}\}$ of $\{Tx_n\}$ with $n(k) > m(k) > k$ such that for every k, $d(Tx_{m(k)}, Tx_{n(k)}) \geq \epsilon$, $d(Tx_{m(k)}, Tx_{n(k)-1}) < \epsilon$. So, we have

$$\begin{aligned} \epsilon &\leq d(Tx_{m(k)},Tx_{n(k)}) \\ &\leq d(Tx_{m(k)},Tx_{n(k)-1})+d(Tx_{n(k)-1},Tx_{n(k)}) \\ &< \epsilon+d(Tx_{n(k)-1},Tx_{n(k)}). \end{aligned}$$

Letting $k \longrightarrow \infty$ and using $d(Tx_{n+1}, Tx_n) \longrightarrow 0$, we have

$$\lim d(Tx_{m(k)},Tx_{n(k)}) = \epsilon = \lim d(Tx_{m(k)},Tx_{n(k)-1}). \tag{2.2}$$

Again,

$$d(Tx_{m(k)},Tx_{n(k)-1}) \leq d(Tx_{m(k)},Tx_{m(k)-1})+d(Tx_{m(k)-1},Tx_{n(k)})+d(Tx_{n(k)},Tx_{n(k)-1}),$$

and

$$d(Tx_{m(k)-1},Tx_{n(k)}) \leq d(Tx_{m(k)-1},Tx_{m(k)})+d(Tx_{m(k)},Tx_{n(k)}).$$

Letting $k \longrightarrow \infty$ in the above two inequalities and using (2.2) we get,

$$\lim d(Tx_{m(k)-1},Tx_{n(k)}) = \epsilon.$$

Also, we have

$$\begin{aligned} \mu(\epsilon) &\leq \mu(d(Tx_{m(k)},Tx_{n(k)})) \\ &\leq \mu(\frac{1}{2}[d(fx_{m(k)},Tx_{n(k)})+d(fx_{n(k)},Tx_{m(k)})]) - \\ &\quad \psi(d(fx_{m(k)},Tx_{n(k)}),d(fx_{n(k)},Tx_{m(k)})) \\ &= \mu(\frac{1}{2}[d(Tx_{m(k)-1},Tx_{n(k)})+d(Tx_{n(k)-1},Tx_{m(k)})]) - \\ &\quad \psi(d(Tx_{m(k)-1},Tx_{n(k)}),d(Tx_{n(k)-1},Tx_{m(k)})). \end{aligned}$$

Taking $k \longrightarrow \infty$, and using the continuity of μ and lower semi-continuity of ψ, we have $\mu(\epsilon) \leq \mu(\frac{1}{2}[\epsilon + \epsilon]) - \psi(\epsilon, \epsilon)$ and consequently $\psi(\epsilon, \epsilon) \leq 0$, which is contradiction since $\epsilon > 0$. Thus $\{Tx_n\}$ is a Cauchy sequence. As $f(X)$ is closed and $fx_n = Tx_{n-1}$, $\{fx_n\}$ is also a Cauchy sequence, there is some $z \in X$ such that $\lim fx_{n+1} = \lim Tx_n = fz$. Since $\{f(x_n)\}$ is a nondecreasing sequence and $\lim fx_{n+1} = fz$, $f(x_n) \le f(z)$, and $f(z) \le f(f(z))$ for every n. Consider

$$\begin{aligned}\mu(d(Tz, fx_{n+1})) &= \mu(d(Tz, Tx_n)) \\ &\le \mu(\frac{1}{2}[d(fz, Tx_n) + d(fx_n, Tz)]) - \psi(d(fz, Tx_n), d(fx_n, Tz)),\end{aligned}$$

letting $n \longrightarrow \infty$, we have

$$\mu(d(Tz, fz)) \le \mu(\frac{1}{2}d(fz, Tz)) - \psi(0, d(fz, Tz))$$

This implies that $d(Tz, fz) = 0$, i.e. $Tz = fz$ and z is a coincidence point of T and f.

Now suppose that T and f are weakly compatible. Let $w = T(z) = f(z)$. Then $T(w) = T(f(z)) = f(T(z)) = f(w)$ and $f(z) \le f(f(z)) = f(w)$. Consider

$$\begin{aligned}\mu(d(T(z), T(w))) &\le \mu(\frac{1}{2}[d(fz, Tw) + d(fw, Tz)]) - \psi(d(fz, Tw), d(fw, Tz)) \\ &= \mu(\frac{1}{2}[d(Tz, Tw) + d(Tw, Tz)]) - \psi(d(Tz, Tw), d(Tw, Tz)) \\ &= \mu(d(Tw, Tz)) - \psi(d(Tz, Tw), d(Tw, Tz)).\end{aligned}$$

This implies that $d(Tz, Tw) = 0$, by the property of ψ. Therefore, $T(w) = f(w) = w$.

Now suppose that the set of common fixed points of T and f is well ordered. We claim that common fixed points of T and f is unique. Assume on contrary that, $Tu = fu = u$ and $Tv = fv = v$ but $u \ne v$. Consider

$$\begin{aligned}\mu(d(u, v)) &= \mu(d(Tu, Tv)) \\ &\le \mu(\frac{1}{2}[d(fu, Tv) + d(fv, Tu)]) - \psi(d(fu, Tv), d(fv, Tu)) \\ &= \mu(\frac{1}{2}[d(u, v) + d(v, u)]) - \psi(d(u, v), d(v, u)) \\ &= \mu(d(u, v)) - \psi(d(u, v), d(u, v)).\end{aligned}$$

This implies that $d(u, v) = 0$, by the property of ψ. Hence $u = v$. Conversely, if T and f have only one common fixed point then the set of common fixed point of f and T being singleton is well ordered. □

If f =identity mapping, then we have the following result.

Corollary 2.2. *Let $(X, \le)$ be a partially ordered set and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Suppose that T is a self mapping on X, T is a monotone nondecreasing mapping and*

$$\mu(d(Tx, Ty)) \le \mu(\frac{1}{2}[d(x, Ty) + d(y, Tx)]) - \psi(d(x, Ty), d(y, Tx)) \tag{2.3}$$

for all $x, y \in X$ for which $x \ge y$ where $\mu : [0, \infty) \longrightarrow [0, \infty)$ is an altering distance function and $\psi : [0, \infty)^2 \longrightarrow [0, \infty)$ is a lower semi-continuous mapping such that $\psi(x, y) = 0$ if and only if $x = y = 0$.

Also suppose that either

(i) $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \longrightarrow z$ in X, then $x_n \le z$, for every n; or

(ii) T is continuous.

If there exists an $x_0 \in X$ with $x_0 \le T(x_0)$, then T has a fixed point.

Moreover, for arbitrary two points $x, y \in X$, there exists $w \in X$ such that w is comparable with both x and y. Then the fixed point of T is unique.

Proof. If (i) holds, then taking f =identity mapping in Theorem 2.1 we get the result.

If (ii) holds then proceeding as in Theorem 2.1 with f =identity mapping, we can prove that $\{Tx_n\}$ is a cauchy sequence, $z = \lim x_{n+1} = \lim T(x_n) = T(\lim x_n) = T(z)$ and hence T has a fixed point.

Let u and v be two fixed points of T such that $u \neq v$. Now, consider the following two cases:

(a) If u and v are comparable. Consider

$$\begin{aligned}
\mu(d(u,v)) &= \mu(d(Tu,Tv)) \\
&\leq \mu(\frac{1}{2}[d(u,Tv)+d(v,Tu)]) - \psi(d(u,Tv),d(v,Tu)) \\
&\leq \mu(\frac{1}{2}[d(u,v)+d(v,u)]) - \psi(d(u,v),d(v,u)) \\
&= \mu(d(u,v)) - \psi(d(u,v),d(u,v)).
\end{aligned}$$

This implies that $d(u,v) = 0$, by the property of ψ. Hence $u = v$.

(b) If u and v are not comparable. Choose an element $w \in X$ comparable with both of them. Then also $u = T^n u$ is comparable with $T^n w$ for each n. Consider

$$\begin{aligned}
\mu(d(u,T^n w)) &= \mu(d(T^n u, T^n w)) \\
&= \mu(d(TT^{n-1}u, TT^{n-1}w)) \\
&\leq \mu(\frac{1}{2}[d(T^{n-1}u,T^n w)+d(T^{n-1}w,T^n u)]) - \psi(d(T^{n-1}u,T^n w),d(T^{n-1}w,T^n u)) \\
&= \mu(\frac{1}{2}[d(u,T^n w)+d(T^{n-1}w,u)]) - \psi(d(u,T^n w),d(T^{n-1}w,u)) \qquad (**) \\
&\leq \mu(\frac{1}{2}[d(u,T^n w)+d(T^{n-1}w,u)])
\end{aligned}$$

and hence we get $d(u,T^n w) \leq d(u,T^{n-1}w)$. This proves that the nonnegative decreasing sequence $\{d(u,T^n w)\}$ is convergent. If $\lim_{n\longrightarrow\infty}\{d(u,T^n w)\} = r$, then, letting $n \longrightarrow \infty$ in $(**)$ and from the continuity of μ and lower semi-continuity of ψ we obtain $\mu(r) \leq \mu(r) - \psi(r,r) \leq \mu(r)$. This gives $\psi(r,r) = 0$ and by our assumption about ψ, $r = 0$. Consequently, $\lim_{n\longrightarrow\infty} d(u,T^n w) = 0$. Analogously, it can be proved that $\lim_{n\longrightarrow\infty} d(v,T^n w) = 0$. Since the limit is unique, we have $u = v$.

□

If $\mu(t) = t$, then we have the following result.

Corollary 2.3. *Let $(X,\leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X,d) is a complete metric space. Suppose that T is a self mapping on X, T is a monotone nondecreasing mapping and*

$$d(Tx,Ty) \leq \frac{1}{2}[d(x,Ty)+d(y,Tx)] - \psi(d(x,Ty),d(y,Tx)) \tag{2.4}$$

for all $x,y \in X$ for which $x \geq y$ where $\mu : [0,\infty) \longrightarrow [0,\infty)$ is an altering distance function and $\psi : [0,\infty)^2 \longrightarrow [0,\infty)$ is a lower semi-continuous mapping such that $\psi(x,y) = 0$ if and only if $x = y = 0$.

Also suppose that either

(i) $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \longrightarrow z$ in X, then $x_n \leq z$, for every n; or

(ii) T is continuous.

If there exists an $x_0 \in X$ with $x_0 \leq T(x_0)$, then T has a fixed point.

Moreover, for arbitrary two points $x, y \in X$, there exists $w \in X$ such that w is comparable with both x and y. Then the fixed point of T is unique.

If $\psi(x,y) = (\frac{1}{2} - k)(x+y)$, $0 < k < \frac{1}{2}$, we have the following result.

Corollary 2.4. [7] *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Suppose that T is a nondecreasing self-mapping of X and T satisfies*

$$d(Tx, Ty) \leq k[d(x, Ty) + d(y, Tx)], \text{ for } x \geq y, \tag{2.5}$$

where $0 < k < \frac{1}{2}$, for all $x, y \in X$. Also suppose either

(i) if $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \longrightarrow z$ in X, then $x_n \leq z$ for every n.

or

(ii) T is continuous.

If there exists an $x_0 \in X$ with $x_0 \leq T(x_0)$, then T has a fixed point.

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