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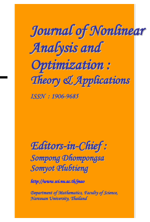

COMMON FIXED POINTS FOR WEAKLY COMPATIBLE MAPS IN INTUITIONISTIC FUZZY METRIC SPACES USING PROPERTY (S-B)

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ABSTRACT. In this note we prove a common fixed point theorem for weakly compatible maps in intuitionistic fuzzy metric spaces using property (S-B), which generalize the results of Kumar and Vats [26], Alaca, Turkoglu and Yildiz [5]

KEYWORDS :Intuitionistic fuzzy metric space, weakly compatible mappings, and (S-B) property.
AMS Subject Classification: 47H10, 54H25

1. INTRODUCTION

The concept of fuzzy sets was introduced initially by Zadeh [45] in 1965. Since, then to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. A fuzzy set A in X is a function with domain X and values in [0, 1]. The concept of fuzzy set corresponds to the degree of nearness between two objects. Deng [9], Erceg [11], Fang [12], George and Veeramani [14], Kaleva and Seikkala [20], Kramosil and Michalek [23] have introduced the concept of fuzzy metric spaces in different ways. Atanassove [6] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Intuitionistic fuzzy sets deals with both degree of nearness and non-nearness. Park [32] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric spaces due to George and Veeramani [14]. Further, Alaca et al. [4] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space, as park with the help of continuous t-norms and continuous t-conorms, as a generalization of fuzzy metric space due to Kramosil and Michalek [24]. Further Coker [8] introduced the concept of intuitionisitc fuzzy topological spaces. Turkoglu et. al. [42] gave a generalization of Jungck's common fixed point

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theorem [17] to intuitionistic fuzzy metric spaces. They first formulated the definition of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of Pant's theorem [31]. Recently, many authors have also studied the fixed point and common fixed point theorems in intuitionistic fuzzy metric space (See [2], [39], [40], [41]). Sharma and Bamboria [38] defined a property (S-B) for self maps and obtained some common fixed point theorems for such mappings under strict contractive conditions. The class of (S-B) maps contains the class of non compatible maps. Kamran [21] obtained some coincidence and fixed point theorems for hybrid strict contractions. Sharma and Sharma [37] proved common fixed point theorem in intuitionistic fuzzy metric spaces under (S-B) property.

2. PRELIMINARIES

The concept of triangular norms (t-norm) and triangular conorms (t- conorm) are known as the axiomatic skeletons that we use for characterization fuzzy intersections and union respectively. These concepts were originally introduced by Menger [29] in study of statistical metric spaces.

Definition 2.1:[36] A binary operation * : $[0,1] \times [0,1] \longrightarrow [0,1]$ is continuous t-norm if * is satisfying the following conditions:

(i) * is commutative and associative;
(ii) * is continuous;
(iii) $a * 1 = a \, \forall \, a \in [0,1]$;
(iv) $a * b \leqslant c * d$ whenever $a \leqslant c$ and $b \leqslant d$; $\forall \, a, b, c, d \in [0,1]$

Definition 2.2:[36] A binary operation $\lozenge$: $[0,1] \times [0,1] \longrightarrow [0,1]$ is continuous t-conorm if $\lozenge$ is satisfying the following conditions:

(i) $\lozenge$ is commutative and associative;
(ii) $\lozenge$ is continuous;
(iii) $a \lozenge 0 = a \, \forall \, a \in [0,1]$;
(iv) $a \lozenge b \leqslant c \lozenge d$ whenever $a \leqslant c$ and $b \leqslant d$; $\forall \, a, b, c, d \in [0,1]$

Alaca et al. [4] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [24] as follows;

Definition 2.3:[4] A 5 tuple $(X, M, N, *, \lozenge)$ is said to an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, $\lozenge$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions;

(i) $M(x,y,t) + N(x,y,t) \leqslant 1 \, \forall \, x, y \in X$ and $t > 0$;
(ii) $M(x,y,t) = 0 \, \forall \, x, y \in X$;
(iii) $M(x,y,t) = 1$ for all $x, y \in X$ and t > 0
(iv) $M(x,y,t) = M(y,x,t) \, \forall \, x, y \in X$ and $t > 0$;
(v) $M(x,y,t) * M(y,z,s) \leqslant M(x,z,t+s)$ for all $x, y, z \in X$ and s, t > 0;
(vi) For all $x, y \in X, M(x,y,.) : [0,\infty) \longrightarrow [0,1]$ is continuous;
(vii) $\lim_{t \longrightarrow \infty} M(x,y,t) = 1$ for all $x, y \in X$ and t > 0;
(viii) $N(x,y,0) = 1$ for all $x, y \in X$;
(ix) $N(x,y,t) = 0$ for all x, y $\in X$ and t > 0 iff $x = y$;
(x) $N(x,y,t) = N(y,x,t)$ for all $x, y \in X$ and $t > 0$;
(xi) $N(x,y,t) \lozenge N(y,z,s) \geqslant N(x,z,t+s)$ for all $x, y, z \in X$ and s, t > 0;

(xii) For all $x, y \in X, N(x, y, .) : [0, \infty) \longrightarrow [0, 1]$ is continuous;
(xiii) $\lim_{t \longrightarrow \infty} N(x, y, t) = 0$ for all $x, y \in X$;

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Example 2.1:[26] Let $X = \{1/n : n \varepsilon N\} u\{0\}$ with * continuous t-norm and $\Diamond$ continuous t-conorm defined by $a * b = ab$ and $a \Diamond b$ = min $\{1, a + b\}$ respectively for all $a, b \in [0, 1]$. For each $t \in (0, \infty)$ and $x, y \in X$, define(M, N) by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|} & \text{if } t > 0. \\ 0 & \text{if } t = 0. \end{cases} \; and \, N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|} & \text{if } t > 0. \\ 1 & \text{if } t = 0. \end{cases}$$

Then $(X, M, N, *, \Diamond)$ is an intuitionistic fuzzy metric space.

Remark 2.1:[27] Every fuzzy metric space $(X, M, *)$is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \Diamond)$ such that t-norm * and t-conorm $\Diamond$ are associated as $x \Diamond y = 1 - ((1 - x) * (1 - y)) \forall x, y \in$ X.

Remark 2.2:[26] An intuitionistic fuzzy metric spaces with continuous t-norm * and continuous t-conorm $\Diamond$ defined by $a * a \geqslant a$ and $(1-a)\Diamond(1-a) \leqslant (1-a) \forall a \in [0, 1]$. Then for all $x, y \in X, M(x, y, *)$ is non- decreasing and $N(x, y, \Diamond)$ is non-increasing.

Alaca, Turkoglu, and Yildiz [4] introduced the following notions:

Definition 2.4: Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Then

(i) a sequence $\{x_n\}$ is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$, $\lim_{n \longrightarrow \infty} M(x_{n+p}, x_n, t) = 1$, $\lim_{n \longrightarrow \infty} N(x_{n+p}, x_n, t) = 0$

(ii) a Sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0, \lim_{n \longrightarrow \infty} M(x_n, x, t) = 1$, $\lim_{n \longrightarrow \infty} N(x_n, x, t) = 0$

Since * and $\Diamond$ are continuous, the limit is uniquely determined from (v) and (xi) of definitions 3 respectively.

Definition 2.5: An intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Turkoglu, Alaca and Yildiz [43] introduced the notions of compatible mappings in intuitionistic fuzzy metric space, akin to the concept of compatible mappings introduced by Jungck [18] in metric spaces.

Definition 2.6: A pair of self mappings (f, g) of a intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be compatible if $\lim_{n \longrightarrow \infty} M(fgx_n, gfx_n, t) = 1$ and $\lim_{n \longrightarrow \infty} N(fgx_n, gfx_n, t = 0$ for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \longrightarrow \infty} fx_n = \lim_{n \longrightarrow \infty} gx_n = z$ for some $z \in X$.

Definition 2.7: A pair of self mappings (f, g) of a intuitionistic fuzzy metric space

$(X, M, N, *, \Diamond)$ is said to be non-compatible if $\lim_{n\longrightarrow\infty} M(fgx_n, gfx_n, t) \neq 1$ or non- existent and $\lim_{n\longrightarrow\infty} N(fgx_n, gfx_n, t) \neq 0$ or non- existent for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\longrightarrow\infty} fx_n = \lim_{n\longrightarrow\infty} gx_n = z$ for some $z \in X$.

In 1998, Jungck and Rhoades [19] introduced the concept of weakly compatible maps as follows.

Definition 2.8: Two self maps f and g are said to be weakly compatible if they commute at coincidence points.

Example 2.2: Let $X = R$ and define $f, g : R \longrightarrow R$ by $fx = x/3$ and $gx = x^2 \,\forall\, x \in R$. Here 0 and $1/3$ are two coincidence points for the maps$f$ and g. Note that $f$and g commute at 0, i.e.$fg(0) = gf(0) = 0$.But $fg(1/3) = f(1/9) = 1/27$ and $gf(1/3) = g(1/9) = 1/81$ and so f and g are not weakly compatible maps on R.

Remark 2.3: Weakly compatible maps need not be compatible.

Aamri and Moutawakil [1] generalized the concept of non compatibility in metric spaces by defining the notion of E.A. property and proved common fixed point theorems. Using this property Sharma and Bamboria [38] defined $(S-B)$ property in fuzzy metric space.

Definition 2.9:[38] A pair of self mappings (S, T) of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to satisfy the $(S-B)$ property if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\longrightarrow\infty} M(Sx_n, Tx_n, t) = 1, \lim_{n\longrightarrow\infty} N(Sx_n, Tx_n, t) = 0$.

Example 2.3: Let $X = [0, \infty)$ consider $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space, where M and N are two fuzzy sets defined by $M(x, y, t) = t/[t + d(x, y)]$ and $N(x, y, t) = d(x, y)/[t + d(x, y)]$ where d is usual metric. Define $T, S : X \longrightarrow [0, \infty)$ by $Tx = x/5$ and $Sx = 2x/5$ for all x in X. Consider $x_n = 1/n$. Now, $\lim_{n\longrightarrow\infty} M(Sx_n, Tx_n, t) = 1, \lim_{n\longrightarrow\infty} N(Sx_n, Tx_n, t) = 0$. Therefore S and T satisfy property $(S - B)$.

Now we state two lemmas which are useful in proving our main results.

Lemma 2.1:[3] Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space and for all $x, y \in X, t > 0$ and if for a number $k \in (0, 1), M(x, y, kt) \geqslant M(x, y, t)$ and $N(x, y, kt) \leqslant N(x, y, t)$ then $x = y$.

Lemma 2.2:[3] Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists a number $k \in (0, 1)$ such that;
$M(y_{n+2}, y_{n+1}, kt) \geqslant M(y_{n+1}, y_n, t), N(y_{n+2}, y_{n+}, kt) \leqslant N(y_{n+1}, y_n, t)$ for all $t > 0$ and $n = 1, 2, \ldots$ then $\{y_n\}$ is a Cauchy sequence in X.

Alaca et al [5] proved the following theorem.

Theorem A: Let A, B, S and T be self maps of a complete intuitionistic fuzzy metric spaces $(X, M, N, *, \Diamond)$ with continuous t-norm * and continuous t-conorm $\Diamond$ defined by $a * a \geqslant a$ and $(1 - a) \leqslant (1 - a) \Diamond (1 - a)$ for all $a \in [0, 1]$ satisfying the following conditions;

$(A.1)$ $A(X) \subset T(X), B(X) \subset S(X)$;
$(A.2)$ S and T are continuous;
$(A.3)$ The pairs $\{A, S\}$ and $\{B, T\}$ are compatible maps;
$(A.4)$ For all$x, y \in X, t > 0$ and $k \in (0, 1)$ such that

$M(Ax, By, kt) \geqslant M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, 2t)* M(Ax, Ty, t)$
$N(Ax, By, kt) \leqslant N(Sx, Ty, t) \lozenge N(Ax, Sx, t) \lozenge N(By, Ty, t) \lozenge N(By, Sx, 2t) \lozenge N(Ax, Ty, t)$.

Then A, B, S and T have a unique common fixed point in X.

Kumar and Vats [26] proved the following.

Theorem B: Let A, B, S and T be self maps of a complete intuitionistic fuzzy metric spaces$(X, M, N, *, \lozenge)$ with continuous t-norm * and continuous t-conorm $\lozenge$ defined by $a* a \geqslant a$ and$(1-a)\lozenge(1-a) \leqslant (1-a)$ for all $a \in [0, 1]$ satisfying the following conditions;

$(B.1)$ $A(X) \subset T(X), B(X) \subset S(X)$
$(B.2)$ $\forall\, x, y \in X, t > 0$ and $k \in (0, 1)$ such that
$M(Ax, By, kt) \geqslant M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, 2t) * M(Ax, Ty, t$
$N(Ax, By, kt) \leqslant N(Sx, Ty, t) \lozenge N(Ax, Sx, t) \lozenge N(By, Ty, t) \lozenge N(By, Sx, 2t) \lozenge N(Ax, Ty, t)$

Then
(i) A and S have a point of coincidence;
(ii) B and T have a point of coincidence.
Moreover, if the pairs$\{A, S\}$ and $\{B, T\}$ are weakly compatible maps, then A, B, S and T have a unique common fixed point in X.

3. MAIN RESULTS

Now, we prove our main result which generalizes the theorems A and B.

Theorem 3.1. *Let A, B, S and T be self maps of a intuitionistic fuzzy metric spaces $(X, M, N, *, \lozenge)$ with continuous t-norm * and continuous t-conorm $\lozenge$ defined by $a * a \geqslant a$ and $(1-a)\lozenge(1-a) \leqslant (1-a)$ for all $a \in [0, 1]$satisfying the following conditions;*
(3.1) A (X) ⊂ T (X), B (X) ⊂ S (X)
(3.2) pairs $\{A, S\}$ and $\{B, T\}$ are weakly compatible.
(3.3) pairs $\{A, S\}$ or $\{B, T\}$ satisfies the property (S-B)
(3.4) for all $x, y \in X, t > 0$ and $k \in (0, 1)$such that
*M (Ax, By, kt) ⩾ M (Sx, Ty, t) * M (Ax, Sx, t) * M (By, Ty, t) * M (By, Sx, 2t) * M (Ax, Ty, t)*
N (Ax, By, kt) ⩽ N (Sx, Ty, t) ◊ N (Ax, Sx, t) ◊ N (By, Ty, t) ◊ N (By, Sx, 2t) ◊ N (Ax, Ty, t)
(3.5) one of A(X), B(X), S(X) or T(X) is a closed subset of X.
Then A, B, S and T have a unique common fixed point in X.

Proof. Suppose that (B, T) satisfies the property $(S - B)$, then there exists a sequence $\{x_n\}$ in X such that $\lim_{n \longrightarrow \infty} Bx_n = \lim_{n \longrightarrow \infty} Tx_n = z$ for some $z \in X$.

Since$B(X) \subset S(X)$ there exists a sequence $\{x_n\} \in$ X such that

$\lim_{n \longrightarrow \infty} Bx_n = \lim_{n \longrightarrow \infty} Sy_n = z$

Now we shall show that $\lim_{n \longrightarrow \infty} Ay_n = z$

From (3.4), we have;

$M(Ay_n, Bx_n, kt) \geqslant M(Sy_n, Tx_n, t) * M(Ay_n, Sy_n, t) * M(Bx_n, Tx_n, t) * M(Bx_n, Sy_n, 2t)$
$* M(Ay_n, Tx_n, t)$
and
$N(Ay_n, Bx_n, kt) \leqslant N(Sy_n, Tx_n, t) \diamondsuit N(Ay_n, Sy_n, t) \diamondsuit N(Bx_n, Tx_n, t) \diamondsuit N(Bx_n, Sy_n, 2t) \diamondsuit N(Ay_n, Tx_n, t)$

Proceeding limit as $n \longrightarrow \infty$, we have;

$\lim_{n \longrightarrow \infty} M(Ay_n, z, kt) \geqslant \lim_{n \longrightarrow \infty} [M(z, z, t) * M(Ay_n, z, t) * M(z, z, t) * M(z, z, 2t)$
$* M(Ay_n, z, t)]$
and
$\lim_{n \longrightarrow \infty} N(Ay_n, z, kt) \leqslant \lim_{n \longrightarrow \infty} [N(z, z, t) \diamondsuit N(Ay_n, z, t) \diamondsuit N(z, z, t) \diamondsuit (z, z, 2t) \diamondsuit N(Ay_n, z, t)]$,

$\lim_{n \longrightarrow \infty} M(Ay_n, z, kt) \geqslant \lim_{n \longrightarrow \infty} [1 * M(Ay_n, z, t) * 1 * 1 * M(Ay_n, z, t)]$
and
$\lim_{n \longrightarrow \infty} N(Ay_n, z, kt) \leqslant \lim_{n \longrightarrow \infty} [0 \diamondsuit N(Ay_n, z, t) \diamondsuit 0 \diamondsuit 0 \diamondsuit N(Ay_n, z, t)]$

It follows that

$\lim_{n \longrightarrow \infty} M(Ay_n, z, kt) \geqslant \lim_{n \longrightarrow \infty} M(Ay_n, z, t)$
and
$\lim_{n \longrightarrow \infty} N(Ay_n, z, kt) \leqslant \lim_{n \longrightarrow \infty} N(Ay_n, z, t)$

and we deduce that $\lim_{n \longrightarrow \infty} Ay_n = z$

Suppose that $S(X)$ is a closed subset of X .Then $z = Su$ for some $u \in X$.
Subsequently, we have,

$\lim_{n \longrightarrow \infty} Ay_n = \lim_{n \longrightarrow \infty} Bx_n = \lim_{n \longrightarrow \infty} Tx_n = \lim_{n \longrightarrow \infty} Sy_n = z = Su.$

Now, we shall show that $Au = Su$

From (3.4) we have,

$M(Au, Bx_n, kt) \geqslant M(Su, Tx_n, t) * M(Au, Su, t) * M(Bx_n, Tx_n, t) * M(Bx_n, Su, 2t)$
$* M(Au, Tx_n, t)$
and
$N(Au, Bx_n, kt) \leqslant N(Su, Tx_n, t) \diamondsuit N(Au, Su, t) \diamondsuit N(Bx_n, Tx_n, t) \diamondsuit N(Bx_n, Su, 2t) \diamondsuit N(Au, Tx_n, t)$

Taking the limit as $n \longrightarrow \infty$, we have,

$M(Au,Su,kt) \geqslant M(Su,Su,t) * M(Au,Su,t) * M(Su,Su,t) * M(Su,Su,2t) * M(Au,Su,t)$
and
$N(Au,Su,kt) \leqslant N(Su,Su,t) \lozenge N(Au,Su,t) \lozenge N(Su,Su,t) \lozenge N(Su,Su,2t) \lozenge N(Au,Su,t)$,

$M(Au,Su,kt) \geqslant 1 * M(Au,Su,t) * 1 * 1 * M(Au,Su,t)$
and
$N(Au,Su,t) \leqslant 0 * N(Au,Su,t) * 0 * 0 * N(Au,Su,t)$,

$M(Au,Su,kt) \geqslant M(Au,Su,t)$
and
$N(Au,Su,kt) \leqslant N(Au,Su,t)$

Therefore by lemma 2.1, we have; $Au = Su$.

i.e. u is a coincidence point of A and S

The weak compatibility of A and S implies that $ASu = SAu$ and then $AAu = ASu = SAu = SSu$ on the other hand since $A(X) \subset T(X)$, there exists a point $v \in X$ such that $Au = Tv$. We claim that $Tv = Bv$.

Suppose that $Tv \neq Bv$. Then (3.4) implies that.

$M(Au,Bv,kt) \geqslant M(Su,Tv,t) * M(Au,Su,t) * M(Bv,Tv,t) * M(Bv,Su,2t) * M(Au,Tv,t)$
and
$N(Au,Bv,kt) \leqslant N(Su,Tv,t) \lozenge N(Au,Su,t) \lozenge N(Bv,Tv,t) \lozenge N(Bv,Su,2t) \lozenge N(Au,Tv,t)$,

$M(Au,Bv,kt) \geqslant M(Au,Au,t) * M(Au,Au,t) * M(Bv,Au,t) * M(Bv,Au,2t) * M(Au,Au,t)$
and
$N(Au,Bv,kt) \leqslant N(Au,Au,t) \lozenge N(Au,Au,t) \lozenge N(Bv,Tu,t) \lozenge N(Bv,Au,2t) \lozenge N(Au,Au,t)$,

$M(Au,Bv,kt) \geqslant 1 * 1 * M(Bv,Au,t) * M(Bv,Au,2t) * 1$
and
$N(Au,Bv,kt) \leqslant 0 \lozenge 0 \lozenge N(Bv,Au,t) \lozenge N(Bv,Au,2t) \lozenge 0$

Thus, it follows that $M(Au,Bv,kt) \geqslant M(Au,Bv,t)$
and
$N(Au,Bv,kt) \leqslant N(Au,Bv,t)$

Therefore by lemma 2.1 we have; $Au = Bv$. Hence $Tv = Bv$.

i.e. v is a coincidence point of B and T.

Thus we have: $Au = Su = Tv = Bv$.

The weak compatibility of B and T implies that

$BTv = TBv$ and $TTv = TBv = BTv = BBv$.

Let us show that Au is a common fixed point of A, B, S and T.

Suppose that $AAu \neq Au$. Then using (3.4) we have;

$M(AAu, Bv, kt) \geqslant M(SAu, Tv, t) * M(AAu, SAu, t) * M(Bv, Tv, t) * M(Bv, SAu, 2t)$
$*M(AAu, Tv, t)$
and
$N(AAu, Bv, kt) \leqslant N(SAu, Tv, t) \Diamond N(AAu, SAu, t) \Diamond N(Bv, Tv, t) \Diamond N(Bv, SAu, 2t) \Diamond N(AAu, Tv, t),$

$M(AAu, Au, kt) \geqslant M(AAu, Au, t) * M(AAu, AAu, t) * M(Bv, Bv, t) * M(Au, AAu, 2t)$
$* M(AAu, Au, t)$
and
$N(AAu, Au, kt) \leqslant N(AAu, Au, t) \Diamond N(AAu, AAu, t) \Diamond N(Bv, Bv, t) \Diamond N(Au, AAu, 2t) \Diamond N(AAu, Au, t),$

$M(AAu, Au, kt) \geqslant M(AAu, Au, t) * 1 * 1 * M(Au, AAu, 2t) * M(AAu, Au, t)$
and
$N(AAu, Au, kt) \leqslant N(AAu, Au, t) \Diamond 0 \Diamond 0 \Diamond N(Au, AAu, 2t) \Diamond N(AAu, Au, t)$

$M(AAu, Au, kt) \geqslant M(AAu, Au, t)$
and
$N(AAu, Au, kt) \leqslant N(AAu, Au, t)$

Therefore by lemma 2.1, we have; $AAu = Au$.

Since $AAu = SAu$, therefore, $Au = AAu = SAu$

i.e. Au is a common fixed point of A and S.

Similarly, we prove that Bv is a common fixed point of B and T. Since $Au = Bv$, we conclude that Au is a common fixed point of A, B, S and T. The proof is similar when $T(X)$ is assumed to be a closed subset of X. The cases in which $A(X)$ or $B(X)$ is a closed subset of X are similar to the cases in which $T(X)$ or $S(X)$ respectively is closed subset of X, since $A(X) \subset T(X)$ and $B(X) \subset S(X)$. Uniqueness follows easily.

□

Theorem 3.2. *Let A, B and S be self maps of a intuitionistic fuzzy metric spaces $(X, M, N, *, \Diamond)$ with continuous t-norm * and continuous t-conorm $\Diamond$ defined by $a * a \geqslant a$ and $(1-a) \Diamond (1-a) \leqslant (1-a)$ for all $a \in [0,1]$ satisfying the following conditions;*
(3.21) A (X) ⊂ S (X), B (X) ⊂ S (X)
(3.22) pairs $\{A, S\}$ and $\{B, S\}$ are weakly compatible.
(3.23) pairs $\{A, S\}$ or $\{B, S\}$ satisfies the property (S-B)
(3.24) for all $x, y \in X, t > 0$ and $k \in (0,1)$ such that
$M^2(Ax, By, kt) \geqslant [M(Ax, Sx, t) * M(By, Sx, t)]^2$
$N^2(Ax, By, kt) \leqslant [N(Ax, Sx, t) \Diamond N(By, Sx, t)]^2$
(3.25) one of A(X), B(X), S(X) or T(X) is a closed subset of X.
Then A, B and S have a unique common fixed point in X.

Proof. Suppose that (B, S) satisfies the property $(S-B)$, then there exists a sequence $\{x_n\}$ in XSuch that $\lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Sx_n = z$ for some $z \in X$.

Since $B(X) \subset S(X)$ there exists a sequence$\{y_n\} \in X$ such that $\lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Sy_n = z$

Now, we shall show that $\lim_{n\longrightarrow\infty} Ay_n = z$

From (3.24), we have; (at $x = y_n, y = x_n$)

$[M(Ay_n, Bx_n, kt)]^2 \geqslant [M(Ay_n, Sy_n, t) * M(Bx_n, Sy_n, t)]^2$

$[N(Ay_n, Bx_n, kt)]^2 \leqslant [N(Ay_n, Sy_n, t) \lozenge N(Bx_n, Sy_n, t)]^2$

Proceeding limit as $n \longrightarrow \infty$, we have;

$[\lim_{n\longrightarrow\infty} M(Ay_n, z, kt)]^2 \geqslant [\lim_{n\longrightarrow\infty} M(Ay_n, z, t) * M(z, z, t)]^2$
and
$[\lim_{n\longrightarrow\infty} N(Ay_n, z, kt)]^2 \leqslant [\lim_{n\longrightarrow\infty} N(Ay_n, z, t) \lozenge N(z, z, t)]^2$

$[\lim_{n\longrightarrow\infty} M(Ay_n, z, kt)]^2 \geqslant [\lim_{n\longrightarrow\infty} M(Ay_n, z, t) * 1]^2$
and
$[\lim_{n\longrightarrow\infty} N(Ay_n, z, kt)]^2 \leqslant [\lim_{n\longrightarrow\infty} N(Ay_n, z, t) \lozenge 0]^2$

Thus, it follows that,

$\lim_{n\longrightarrow\infty} M(Ay_n, z, kt) \geqslant \lim_{n\longrightarrow\infty} M(Ay_n, z, t)$
and
$\lim_{n\longrightarrow\infty} N(Ay_n, z, kt) \leqslant \lim_{n\longrightarrow\infty} N(Ay_n, z, t)$

and we deduce that $\lim_{n\longrightarrow\infty} Ay_n = z$.

Suppose that $S(X)$ is a closed subset of X. then $z = Su$. For some $u \in X$, subsequently we have;

$\lim_{n\longrightarrow\infty} Ay_n = \lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Sy_n = z = Su$

Now, we shall show that $Au = Su$.

From (3.24) we have (at $x = u, y = x_n$)

$[M(Au, Bx_n, kt)]^2 \geqslant [M(Au, Su, t) * M(Bx_n, Su, t)]^2$
and
$[N(Au, Bx_n, kt)]^2 \leqslant [N(Au, Su, t) \lozenge N(Bx_n, Su, t)]^2$

Taking the limit as $n \longrightarrow \infty$, we have;

$[M(Au, Su, kt)]^2 \geqslant [M(Au, Su, t) * M(Su, Su, t)]^2$
and
$[N(Au, Su, kt)]^2 \leqslant [N(Au, Su, t) \lozenge N(Su, Su, t)^2$

$[M(Au, Su, kt)]^2 \geqslant [M(Au, Su, t) * 1]^2$

and
$[N(Au, Su, kt)]^2 \leqslant [N(Au, Su, t) \diamondsuit 0]^2$

Thus, it follows that,

$M(Au, Su, kt) \geqslant M(Au, Su, t)$
and
$N(Au, Su, kt) \leqslant N(Au, Su, t)$

Therefore, by lemma 2.1, we have; $Au = Su$.

i.e. u is a coincidence point of A and S.

The weak compatibility of A and S implies that $ASu = SAu$ and then $AAu = ASu = SAu = SSu$. On the other hand, since $A(X) \subset S(X)$, there exist a point $v \in X$ such that $Au = Sv$. We claim that $Sv = Bv$.

i.e. v is a coincidence point of S and B.

Using (3.24), we have;

$[M(Au, Bv, kt)]^2 \geqslant [M(Au, Su, t) * M(Bv, Su, t)]^2$
and
$[N(Au, Bv, kt)]^2 \leqslant [N(Au, Su, t) \diamondsuit N(Bv, Su, t)]^2$

$[M(Au, Bv, kt)]^2 \geqslant [M(Au, Au, t) * M(Bv, Au, t)]^2$
and
$[N(Au, Bv, kt)]^2 \leqslant [N(Au, Au, t) \diamondsuit N(Bv, Au, t)]^2$

$[M(Au, Bv, kt)]^2 \geqslant [1 * M(Au, Bv, t)]^2$
and
$[N(Au, Bv, kt)]^2 \leqslant [0 \diamondsuit N(Au, Bv, t)]^2$

Thus, it follows that,

$M(Au, Bv, kt) \geqslant M(Au, Bv, t)$
and
$N(Au, Bv, kt) \leqslant N(Au, Bv, t)$

Therefore, by lemma 2.1, we have; $Au = Bv$

Thus $Au = Su = Sv = Bv$.

The weak compatibility of B and S implies $BSv = SBv$ and then$BBv = BSv = SBv = SSv$.

Let us show that Au is a common fixed point of A, B and S.

In view of (3.24), it follows that

$[M(AAu, Bv, kt)]^2 \geqslant [M(AAu, SAu, t) * M(Bv, SAu, t)]^2$
and
$[N(AAu, Bv, kt)]^2 \leqslant [N(AAu, SAu, t) \lozenge N(Bv, SAu, t)]^2$

$[M(AAu, Bv, kt)]^2 \geqslant [M(AAu, AAu, t) * M(Au, AAu, t)]^2$
and
$[N(AAu, Bv, kt)]^2 \leqslant [N(AAu, AAu, t) \lozenge N(Au, AAu, t)]^2$

$[M(AAu, Bv, kt)]^2 \geqslant [1 * M(Au, AAu, t)]^2$
and
$[N(AAu, Bv, kt)]^2 \leqslant [0 \lozenge N(Au, AAu, t)]^2$

That, it follows that,

$M(AAu, Au, kt) \geqslant M(Au, AAu, t)$
and
$N(AAu, Au, kt) \leqslant N(Au, AAu, t)$

Therefore, by lemma 2.1, we have; $AAu = Au$.

Thus $AAu = Au = SAu$ and Au is a common fixed point of A and S.

Similarly, we prove that Bv is a common fixed point of B and S.

Since $Au = Bv$, we conclude that Au is a common fixed point of A, B and S.

If $Az = Bz = Sz = z$ and $Aw = Bw = Sw = w$, then by (3.24), we have;

$[M(Az, Bw, kt)]^2 \geqslant [M(Az, Sz, t) * M(Bw, Sz, t)]^2$
and
$[N(Az, Bw, kt)]^2 \leqslant [N(Az, Sz, t) \lozenge N(Bw, Sz, t)]^2$

$[M(z, w, kt)]^2 \geqslant [M(z, z, t) * M(w, z, t)]^2$
and
$[N(z, w, kt)]^2 \leqslant [N(z, z, t) \lozenge N(w, z, t)]^2$

$[M(z, w, kt)]^2 \geqslant [1 * M(z, w, t)]^2$
and
$[N(z, w, kt)]^2 \leqslant [0 \lozenge N(z, w, t)]^2$

Thus, it follows that,

$M(z, w, kt) \geqslant M(z, w, t)$
and
$N(z, w, kt) \leqslant N(z, w, t)$

Therefore, by lemma 2.1, we have;$z = w$.

Hence the common fixed point is unique.

This completes the proof of the theorem.

□

References

1. Aamri, M. and Moutawakil, D. El: Some new common fixed point theorems under strict contractive conditions, J. Math. Anal. Appl., 270(2002), 181-188.
2. Abbas, M., Ali, B. Sintunavarat, W. and Kumam, P.:Tripled fixed point and Tripled coincidence point theorems in intuitionistic fuzzy normed spaces, Fixed point Theory and Applications 2012,2012:187.
3. Alaca, C., Altun, I. and Turkoglu,D.: On compatible mappings of type (I) and type (II) in intuitionistic fuzzy metric spaces, Commun. Korean Math. Soc., 23 (2008), No. 3. 427-446.
4. Alaca, C., Turkoglu, D. and Yildiz, C.: Fixed points in intuitionistic fuzzy metric spaces, Chaos, Solitons and Frectals 29(2006), 1073- 1078.
5. Alaca, C., Turkoglu,D. and Yildiz,C.: Common Fixed Points of Compatible Maps in Intuitionistic Fuzzy Metric Spaces, Southeast Asian Bulletin of Mathematics, 32(2008), 21-33.
6. Atanassov, K.: Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1986), 87- 96.
7. Banach, S.: Theorie les operations linearies, Manograie Mathematyezne Warsaw Poland, 1932.
8. Coker, D.: An inroduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems 88 (1997), 81-99.
9. Deng, Z. K. : Fuzzy pseudo metric spaces, J. Math. Anal. Appl., 86(1982), 74-95.
10. Edelstein, M. : On fixed and periodic points under contraction mappings, J. London Math. Soc., 37(1962), 74-79.
11. Erceg, M. A.: Metric spaces in fuzzy set theory, J. Math. Anal. Appl., 69(1979), 205-230.
12. Fang, J. X.: On fixed point theorems in fuzzy metric spaces, Fuzzy Sets and Systems, 46(1992), 107-113.
13. George, A. and Veeramani, P.: On some results in fuzzy metric spaces, Fuzzy Sets and Systems, 64(1994), 395-399.
14. George, A. and Veeramani, P.: On some results of analysis for fuzzy smetric spaces, Fuzzy Sets and Systems, 90(1997), 365-368.
15. Grabiec, M.: Fixed point in fuzzy metric space, Fuzzy Sets and Systems, 27(1988), 385-389.
16. Gregori, V., Romaguera, S. and Veeramani, P: A note on intuitionistic fuzzy metric spaces, Chaos, Solitions and Fractals 28(2006), 902- 905.
17. Jungck, G.: Commuting mappings and fixed points, Amer. Math. Monthly, 83(1976), 261-263.
18. Jungck, G.: Compatible mappings and common fixed points, Internat. J. Math. Math. Sci., 9(1986), 771-779.
19. Jungck G., and Rhoades, B. E.: Fixed point for set valued functions without continuity, Ind. J. Pure Appl. Maths., 29(3)(1998), 227-238.
20. Kaleva, O. and Seikkala, S.: On fuzzy metric spaces, Fuzzy Sets and Systems, 12(1984), 215-229.
21. Kamran, T.: Coincidence and fixed points for hybrid strict contractions, J. Math. Anal. Appl., 299(2004), 235-241.
22. Klement, E.P., Mesiar, R. and Pap, E.: A characterization of the ordering of continuous t-norms, Fuzzy Sets and Systems 86 (1997), 189-195.
23. Klement, E.P., Mesiar, R. and Pap, E.: Triangular Norms, Kluwer Academic Publ., Trends in Logic 8, and Dordrecht. 2000.
24. Kramosil, I. and Michalek, J.: Fuzzy metric and statistical metric spaces, Kybernetika, 11(1975), 336-344.
25. Kumar,S.: Common fixed points theorems for expansion mappings in various spaces, Acta Math. Hungerica, 118(1-2) 92008), 9-28.
26. Kumar,S. and Vats,R.: Common Fixed points for weakly compatible maps in intuitionistic fuzzy metric spaces, Advances in fuzzy Mathematics, Volume 4, Number 1 (2009), pp. 9-22 © Research India Publications.
27. Lowen,R. : Fuzzy Set Theory, Kluwer Academic Publ., Dordrecht 1996.
28. Mishra, S. N., Sharma, N. and Singh, S. L.: Common fixed points of maps in fuzzy metric spaces. Internat. J. Math. Math. Sci., 17(1994), 253-258.
29. Menger, K.: Statistical Metric, Proc. Nat. Acad. Sci. U. S. A., 28(1942), 535-537.
30. Mihet, D.: On Fuzzy contractive mappings in fuzzy metric spaces, Fuzzy Sets and Systems, 158(2007), 915-921.

31. Pant, R.P.: Common fixed points of non commuting mappings, J. Math. Anal. Appl., 188(1994), 436-440.
32. Park, J. H.: Intuitionistic fuzzy metric spaces, Chaos, Solitions and Fractals 22(2004), 1039-1046.
33. Park, J.S., Kwun,Y.C. and Park,J.H.: A fixed pint theorem in the intuitionistic fuzzy metric spaces, Far East J. Math. Sci., 16 (2005), 137-149.
34. Pathak, H. K., Cho, Y. J. and Kang, S. M. : Remarks on R commuting maps and common fixed point theorems, Bull. Korean Math. Soc. 34 (1997), 247-257.
35. Sadati, R. and Park, J. H.: On the intuitionistic topological spaces, Chaos, Solutions Fractals 27(2006), 331-344.
36. Schweizer, B. and Sklar, A.: Statistical metric spaces, Pacific. J. Math., 10(1960), 313-334.
37. Sharma, Praveen and Sharma, S.: Common fixed point theorem in intuitionistic fuzzy metric space under (S-B) property, J. of Non linear Analysis and Optimization, Vol.3, No.2 (2012), 161-169.
38. Sharma, Sushil and Bamboria, D.: Some new common fixed point theorems in fuzzy metric space under strict contractive conditions, J. Fuzzy Math., Vol. 14, No.2 (2006), 1-11.
39. Sintunavarat,W. ,Cho,Y.J. and Kumam, P: Coupled coincidence point theorems for contractions without commutative condition in intuitionistic fuzzy normed spaces ,Fixed point Theory and Applications 2011,2011:81.
40. Sintunavarat,W. and Kumam, P.: Fixed point theorems for a generalized intuitionistic fuzzy contraction in intuitionistic fuzzy metric spaces,Thai journal of Mathematics 10(1) (2012) 123-135.
41. Sintunavarat,W. Chouhan ,S. and Kumam, P.: Some fixed point results in modified intuitionistic fuzzy metric spaces and applications to integral type , Afrika matematika (to be appear) DOI:10.1007/s13370012-0128-0
42. Turkoglu, D., Alaca, C., Cho, Y. J. and Yildiz, C.: Common fixed point theorems in intuitionistic fuzzy metric spaces, J. Appl. Math. and Computing 22, No.1-2(2006), 411-424.
43. Turkoglu, D., Alaca, C. and Yildiz, C.: Compatible maps and compatible maps of type (α) and (β) in intuitionistic fuzzy metric spaces, Demonstratio Math. 39, No.3 (2006), 671-684.
44. Turkoglu,D., Altun ,I.and Cho,Y.J.: Common fixed points of compatible mappings of type (I) and (II) in fuzzy metric spaces, J. Fuzzy Math., 15(2007), 435-448.
45. Zadeh, L.A.: Fuzzy Sets, Inform Contr., 8(1965), 338-353.