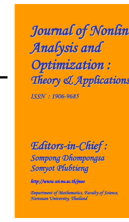
Journal of **N**onlinear **A**nalysis and **O**ptimization
Vol. 4, No. 2, (2013), 205-218
ISSN : 1906-9605
http://www.math.sci.nu.ac.th



EXISTENCE OF NONLINEAR NEUTRAL IMPULSIVE INTEGRODIFFERENTIAL EVOLUTION EQUATIONS OF SOBOLEV TYPE WITH TIME VARYING DELAYS

B. RADHAKRISHNAN*, A. MOHANRAJ†, V. VINOBA‡

* Department of Applied Mathematics & Computational Sciences, PSG College of Technology, Coimbatore- 641004, Tamil Nadu, India

† Department of Science and Humanities (Mathematics), SVS College of Engineering, Coimbatore-642109,Tamil Nadu, India

‡ Department of Mathematics, K.N.Govt. Arts College for Women, Thanjavur-613007,India

ABSTRACT. In this paper, we prove the existence of solutions for nonlinear neutral impulsive evolution integrodifferential equations of Sobolev type with time varying delays. The results are obtained by using semigroup theory and the Monch's fixed point theorem. An application of the same problem is discussed. An example is provided to illustrate the theory.

KEYWORDS: Existence; Neutral differential equation; Impulsive differential equation; Measure of noncompactness; Fixed point theorem.
AMS Subject Classification: 34A37, 47D06, 47H10, 74H20, 34K40.

1. INTRODUCTION

A large class of scientific and engineering problems is modelled by partial differential equations, integral equations or coupled ordinary and partial differential equations which can be described as differential equations in infinite dimensional spaces using semigroups. In general functional differential equations or evolution equations serve as an abstract formulations of many partial differential equations which arise in problems connected with heat-flow in materials with memory, viscoelasticity and many other physical phenomena. Using the method of semigroups, various solutions of nonlinear and semilinear evolution equations have been discussed by Pazy [27] and the nonlocal problem for the same equations has been first studied by Byszewskii [11–13]. Because it is demonstrated that the nonlocal problems have better effects in applications than the classical Cauchy problems. Such problems with nonlocal conditions have been extensively studied in literature [1, 5, 6, 31]. Balachandran et al. [8] studied the nonlocal Cauchy problem for delay integrodifferential equations of Sobolev type in Banach spaces. Bahuguna and

* *Corresponding author.*
Email address : radhakrishnanb1985@gmail.com (B. Radhakrishnan).
Article history : Received 8 March 2013. Accepted 17 October 2013.

Shukla [9] established the approximation of solutions to nonlinear Sobolev type evolution equations. Showalter [30] established the existence of solutions of semilinear evolution equations of Sobolev type in Banach spaces. This type of equations arise in various applications such as in the flow of fluid through fissured rocks, thermodynamics, and shear in second-order uids. For more details, we refer the reader to [10, 21, 22].

A delay differential equation is a special type of functional differential equation. Delay differential equations are similar to ordinary differential equation, but their evolution involves past values of the state variable. Time delay is inherently the character of most dynamical systems to some extent. Particularly the delays in many engineering systems such as power systems are often time-varying and sometimes vary violently with time. Time delays are frequently encountered in various engineering systems such as aircraft, long transmission lines in pneumatic models and chemical or process control systems. These delays may be the source of instability and lead to serious deterioration in the performance of closed loop systems. Theory of neutral differential equations has been studied by several authors in Banach spaces [15, 16, 18–20].

Consequently, it is natural to assume, in modeling these problems, that these perturbations act instantaneously, that is in the form of impulses. The theory of impulsive differential equations has become an active area of investigation due to their applications in the field such as mechanics, electrical engineering, medicine biology and so on. However, one may easily visualize that abrupt changes such as shock, harvesting and disasters may occur in nature. These phenomena are short time perturbations whose duration is negligible in comparison with the duration of the whole evolution process. Consequently, it is natural to assume, in modeling these problems, that these perturbations act instantaneously, that is in the form of impulses. The theory of impulsive differential equation [23, 26, 29] is much richer than the corresponding theory of differential equations without impulsive effects. The impulsive condition

$$\Delta u(t_i) = u(t_i^+) - u(t_i^-) = I_i(u(t_i^-)),\ i = 1,\ 2, \ldots, m,$$

is a combination of traditional initial value problems and short-term perturbations whose duration is negligible in comparison with the duration of the process. Lin and Liu [24] discussed the iterative methods for the solution of impulsive functional differential systems.

Measures of noncompactness are a very useful tool in many branches of mathematics. They are used in the fixed point theory, linear operators theory, theory of differential and integral equations and others [3]. There are two measures which are the most important ones. The Kuratowski measure of noncompactness $\sigma(X)$ of a bounded set X in a metric space is defined as infimum of numbers $r > 0$ such that X can be covered with a finite number of sets of diameter smaller than r. The Hausdorff measure of noncompactness $\chi(X)$ defined as infimum of numbers $r > 0$ such that X can be covered with a finite number of balls of radii smaller than r. There exist many formula on $\chi(X)$ in various spaces [3, 4]. The notion of a measure of weak compactness was introduced by De Blasi [14] and was subsequently used in numerous branches of functional analysis and the theory of differential and integral equations. Several authors have studied the measures of noncompactness in Banach spaces [2, 4].

Motivated by the above literature, the goal of this paper is to use the fixed point theorem to obtain the existence of mild solution of sobolev type nonlinear neutral impulsive integrodifferential evolution equations with time varying delays.

2. PRELIMINARIES

In this section, we recall some definitions, notations and results that we need in the sequel. Throughout this paper, $(X, \|\cdot\|)$ is a Banach space and $A(t)$ generates the evolution operator in X. Also $A(t),\ t \in I$ is closed linear operator defined on a common domain $\mathcal{D} := D(A(t))$, which is dense in X.

The purpose of this paper is to prove the existence of mild solutions for a nonlinear impulsive neutral delay integrodifferential equation of Sobolev type with nonlocal conditions of the form

$$\begin{aligned} & \frac{d}{dt}\left[Bx(t) + e(t, x(\sigma_1(t)))\right] + A(t)x(t) \\ & \quad = f(t, x(\sigma_2(t))) + \int_0^t k(t,s)h(s, x(\sigma_3(s)))ds,\ t \in I,\ t \neq t_k, \quad (2.1) \end{aligned}$$

$$x(0) + g(x) = x_0, \quad (2.2)$$

$$\Delta x(t_k) = I_k(x_{t_k}), \quad k = 1, 2, \ldots, m \quad (2.3)$$

where $A(t), B$ are two closed operators such that $-A(t)\ B^{-1}$ generates the strongly continuous semigroup of bounded linear operators $U(t,s)$ in a Banach space X and $I = [0, a]$. The nonlinear operators $f : [0, a] \times X \to X, k : [0, a] \times [0, a] \to \mathcal{R}$, $h : [0, a] \times X \to X$, $e : [0, a] \times X \to X$, $g : \mathcal{PC}([0, a], X) \to D(B)$ and the delay $\sigma_i(t) \leq t, i = 1, 2, 3$ are given appropriate functions; $x(t_k^+)$ and $x(t_k^-)$ represent the right and left limits of $x(t)$ at $t = t_k$, for $0 = t_0 < t_1 ... < t_k < t_{k+1} = a$.

Let $(X, \|.\|)$ be a real Banach space. $\{A(t) : t \in \mathbb{R}\}$ is a family of closed linear operators defined on a common domain $\mathcal{D}$ which is dense in X and we assume that the linear non-autonomous system

$$\begin{aligned} u'(t) &= A(t)u(t), \quad s \leq t \leq a, \\ u(s) &= x \in X, \end{aligned} \quad (2.4)$$

has associated evolution family of operators $\{U(t,s) : 0 \leq s \leq t \leq a\}$. In the next definition, $\mathcal{L}(X)$ is a space of bounded linear operator from X into X endowed with the uniform convergence topology.

Definition 2.1. A family of operators $\{U(t,s) : 0 \leq s \leq t \leq a\} \subset \mathcal{L}(X)$ is called a evolution family of operators for (2.4), if the following properties hold:

(i) $U(t,s)U(s,\tau) = U(t,\tau)$ and $U(t,t)x = x$, for every $s \leq \tau \leq t$ and all $x \in X$;
(ii) For each $x \in X$, the functions for $(t,s) \longrightarrow U(t,s)x$ is continuous and $U(t,s) \in \mathcal{L}(X)$ for every $t \geq s$ and
(iii) For $0 \leq s \leq t \leq a$, the function $t \longrightarrow U(t,s)$, for $(s,t] \in \mathcal{L}(X)$, is differentiable with $\frac{\partial}{\partial t}U(t,s) = A(t)U(t,s)$.

We denote by $\mathcal{PC}([0,a], X)$ the space of $X-$ valued continuous functions on $[0, a]$ with the norm $\|x\| = \sup\{\|x(t)\|, t \in [0, a]\}$ and by $\mathcal{L}^1([0, a], X)$ the space of $X-$valued Bochner integrable functions on $[0, a]$ with the norm

$$\|f\|_{\mathcal{L}^1} = \int_0^a \|f(t)\|dt.$$

Let us recall the following definition.

Definition 2.2. A continuous solution $x(t)$ of the integral equation

$$\begin{aligned} x(t) &= B^{-1}U(t,0)B[x_0 - g(x)] + B^{-1}U(t,0)e(0, x(0)) \\ &\quad - B^{-1}e(t, x(\sigma_1(t)) + \int_0^t U(t,s)A(s)B^{-1}e(s, x(\sigma_1(s))ds \end{aligned}$$

$$+\int_0^t U(t,s)B^{-1}\left[f(s,x(\sigma_2(s)))+\int_0^s k(s,\tau)h(\tau,x(\sigma_3(\tau)))d\tau\right]ds$$
$$+\sum_{0<t_i<t} B^{-1}U(t,t_k)I_kx(t_k) \quad (2.5)$$

is said to be a mild solution of problem $(2.1)-(2.3)$ on $[0,a]$.

To prove our main theorem we assume certain conditions on the operators $A(t)$ and B. Let X and Y be Banach spaces with norm $|\cdot|$ and $\|\cdot\|$ respectively. The operators $A(t):D(A(t))\subset X\longrightarrow Y$ and $B:D(B)\subset X\longrightarrow Y$ satisfy the following hypothesis:

(A1) $A(t)$ and B are closed linear operators;
(A2) $D(B)\subset D(A(t))$ and B is bijective;
(A3) $B^{-1}:Y\to D(B)$ is continuous.

The hypothesis $(\mathbf{A1})-(\mathbf{A3})$ and the closed graph theorem imply the boundedness of the linear operator $A(t)B^{-1}:X\to X$ and $-A(t)B^{-1}$ generates a uniformly continuous evolution operators $U(t,s), t\geq 0$, of bounded linear operators on Banach space X.

Next, we introduce the Hausdorff's measure of noncompactness $\psi(\cdot)$ defined on each bounded subset E of Banach space Y by

$$\psi(B)=\inf\{\epsilon>0;\ B\ has\ a\ finite\ \epsilon-\ net\ in\ Y\}.$$

Lemma 2.3. *[3] Let Y be a real Banach space and $C,E\subseteq Y$ be bounded, with the following properties:*

- (i) *C is pre-compact if and only if $\psi_Y(B)=0$.*
- (ii) *$\psi_Y(C)=\psi_Y(\bar{C})=\psi_Y(conC)$, where $\bar{C}$ and $conC$ mean the closure and convex hull of C respectively.*
- (iii) *$\psi_Y(C)\leq\psi_Y(E)$, where $C\subseteq E$.*
- (iv) *$\psi_Y(C+E)\leq\psi_Y(C)+\psi_Y(E)$, where $C+E=\{x+y:\ x\in C,\ y\in E\}$.*
- (v) *$\psi_Y(C\cup E)\leq\max\{\psi_Y(C),\ \psi_Y(E)\}$.*
- (vi) *$\psi_Y(\lambda C)\leq|\lambda|\psi_Y(C)$, for any $\lambda\in\mathcal{R}$.*
- (vii) *If the map $\mathcal{F}:D(\mathcal{F})\subseteq Y\to Z$ is Lipschitz continuous with constant r,then $\psi_Z(\mathcal{F}B)\leq r\psi_Y(B)$, for any bounded subset $B\subseteq D(\mathcal{F})$, where Z be a Banach space.*

Before we prove the existence results, we need the following Lemmas.

Lemma 2.4. *[3] If $\mathbb{W}\subseteq\mathcal{PC}([0,a],X)$ is bounded, then $\psi(\mathbb{W}(t))\leq\psi_c(\mathbb{W})$ for all $t\in[0,a]$, where $\mathbb{W}(t)=\{u(t);u\in\mathbb{W}\}\subseteq X$. Furthermore if $\mathbb{W}$ is equicontinuous on $[0,a]$, then $\psi(\mathbb{W}(t))$ is continuous on $[0,a]$ and $\psi_c(\mathbb{W})=\sup\{\psi(\mathbb{W}(t)),\ t\in[0,a]\}$.*

Lemma 2.5. *[17, 25] If $\{u_n\}_{n=1}^{\infty}\subset\mathcal{L}^1([0,a],X)$ is uniformly integrable, then the function $\psi(\{u_n(t)\}_{n=1}^{\infty})$ is measurable and*

$$\psi\Big\{\Big(\int_0^t u_n(s)ds\Big)_{n=1}^{\infty}\Big\}\leq 2\int_0^t\psi(\{u_n(s)\}_{n=1}^{\infty})ds. \quad (2.6)$$

The following fixed point theorem, a nonlinear alternative of Monch type, plays a key role in our existence of mild solutions for nonlocal Cauchy problem $(2.1)-(2.3)$.

Theorem 2.6. *Let Y be a Banach space, U an open subset of Y and $0\in U$. Suppose that $F:\overline{U}\to Y$ is a continuous map which satisfies Monch's condition (that is, if $D\subseteq\overline{U}$ is countable and $D\subseteq\overline{co}(0\cup F(D))$, then $\overline{D}$ is compact) and assume that*

$$x\neq\lambda F(x),\quad for\ \ x\in\partial U\ \ and\ \ \lambda\in(0,1) \quad (2.7)$$

holds. Then F has a fixed point in $\overline{U}$.

3. MAIN RESULTS

In this section, we study the existence of mild solutions of neutral impulsive evolution integrodifferential equations of Sobolev type.

To prove our existence results, we assume the following hypotheses:

(M1) $A(t)$ generates a family of evolution operator $U(t,s)$, when $t > s > 0$, of C_0- semigroups on X and there exists a constant $M > 0$ such that

$$\|U(t,s)\| \leq M, \quad \text{for } \ 0 \leq s \leq t \leq a.$$

(M2) (i) The nonlinear function $e : [0,a]\times X \to X$, for a.e $t \in [0,a]$, the function $e(\cdot, x)$ is continuous and for all $x \in X$, the function $e(\cdot, x) : [0,a] \to X$ is measurable, for all $x \in X$.

(ii) There exist functions $\phi_0, \phi_1, \phi_2 \in \mathcal{L}^1([0,a], \mathcal{R}^+)$ and nondecreasing continuous functions $\Omega_e, \Omega_{Ae} : \mathcal{R}^+ \to \mathcal{R}^+$ such that for every $x \in X$, we have

$$\begin{aligned} \|e(t,x)\| &\leq \phi_1(t)\ \Omega_e\|x\|, \ a.e. \ t \in [0,a] \\ \|A(t)e(t,x)\| &\leq \phi_2(t)\ \Omega_{Ae}\|x\|, \ a.e. \ t \in [0,a] \\ \|e(0,x)\| &\leq \phi_0, \ \ a.e. \ t \in [0,a]. \end{aligned}$$

(iii) There exist functions $\gamma_0, \gamma_e, \gamma_{Ae} \in \mathcal{L}^1([0,a], \mathcal{R}^+)$ such that for every bounded $D \subset X$, we have

$$\begin{aligned} \psi(e(t,D)) &\leq \gamma_e(t)\psi(D), \ \ a.e. \ t \in [0,a] \\ \psi(A(t)e(t,D)) &\leq \gamma_{Ae}(t)\psi(D), \ \ a.e. \ t \in [0,a] \\ \psi(e(0,D)) &\leq \gamma_0, \ \ a.e. \ t \in [0,a]. \end{aligned}$$

(M3) (i) The nonlinear function $f : [0,a]\times X \to X$, for a.e $t \in [0,a]$, the function $f(\cdot, x)$ is continuous and for all $x \in X$, the function $f(\cdot, x) : [0,a] \to X$ is measurable for all $x \in X$.

(ii) There exists a function, $\phi_3 \in \mathcal{L}^1([0,a], \mathcal{R}^+)$ and a nondecreasing continuous function $\Omega_f : \mathcal{R}^+ \to \mathcal{R}^+$ such that for every $x \in X$, we have

$$\|f(t,x)\| \leq \phi_3(t)\Omega_f\|x\|, \ \ a.e \ t \in [0,a].$$

(iii) There exists a function, $\gamma_f \in \mathcal{L}^1([0,a], \mathcal{R}^+)$ such that for every bounded $D \subset X$, we have

$$\psi(f(t,D)) \leq \gamma_f(t)\psi(D), \quad a.e \ t \in [0,a].$$

(M4) (i) The nonlinear function $h : [0,a]\times X \to X$, for a.e $t \in [0,a]$, the function $h(\cdot, x)$ is continuous and for all $x \in X$, the function $f(\cdot, x) : [0,a] \to X$ is strongly measurable, for all $x \in X$.

(ii) There exists a function, $\phi_4 \in \mathcal{L}^1([0,a], \mathcal{R}^+)$ and a nondecreasing continuous function $\Omega_h : \mathcal{R}^+ \to \mathcal{R}^+$ such that for every $x \in X$, we have

$$\|h(t,x)\| \leq \phi_4(t)\Omega_h\|x\|, \ \ a.e \ t \in [0,a].$$

(iii) There exists a function, $\gamma_h \in \mathcal{L}^1([0,a], \mathcal{R}^+)$ such that for every bounded $D \subset X$, we have

$$\psi(h(t,D)) \leq \gamma_h(t)\psi(D), \ \ a.e \ t \in [0,a].$$

(M5) The function $k : [0,a] \times [0,a] \to R$ is measurable function such that there exist a constant K such that

$$\|k(t,s)\| \leq K, \text{ for } s,t \in I.$$

(M6) (i) $I_k : X \longrightarrow X$ is continuous. There exists a nondecreasing continuous function $\Omega_I : \mathcal{R}^+ \to \mathcal{R}^+$ such that for every $x \in X$, we have

$$\|I_k(x(t_k))\| \leq \Omega_I \|x\|, \text{ where } k = 1,2,3,\dots,m.$$

(ii) There exists a function, $\gamma_I \in \mathcal{L}^1([0,a], \mathcal{R}^+)$ such that, for every bounded $D \subset X$, we have

$$\psi(I_k(D)) \leq \gamma_I(t)\psi(D), \quad k = 1,2\dots,m.$$

(M7) The function $g : \mathcal{PC}([0,a],X) \to D(B)$ is continuous compact map such that $\|g(x)\| \leq c\|x\| + d$, for all $x \in \mathcal{PC}([0,a],X)$, for some positive constants c and d.

Now, we give the existence results for $(2.1) - (2.3)$.

Theorem 3.1. *Assume that the conditions $(M1) - (M7)$ are satisfied. Then, for every $x_0 \in D(B)$ the impulsive nonlocal problem $(2.1) - (2.3)$ has at least one mild solution $[0,a]$ provided that there exists a constant $\mathcal{N} > 0$ with*

$$\frac{(1-\alpha\beta Mc)\mathcal{N}}{\alpha\beta M(d+\|x_0\|)+\alpha\phi_1\Omega_e(\mathcal{N})+\alpha M[\phi_0+\phi_2\Omega_{Ae}(\mathcal{N})+\phi_3\Omega_f(\mathcal{N})+K\phi_4\Omega_h(\mathcal{N})+\Omega_I(\mathcal{N})]} > 1 \tag{3.1}$$

and that

$$2\alpha[\|\gamma_e\| + M\{\|\gamma_0\| + \|\gamma_{Ae}\| + \|\gamma_f\| + K\|\gamma_h\| + \|\gamma_I\|\}] < 1. \tag{3.2}$$

Proof. We consider the operator $\mathcal{F} : \mathcal{PC}([0,a],X) \to \mathcal{PC}([0,a],X)$ defined by

$$(\mathcal{F}x)(t) = (\mathcal{F}_1 x)(t) + (\mathcal{F}_2 x)(t) \tag{3.3}$$

with

$$(\mathcal{F}_1 x)(t) = B^{-1}U(t,0)B[x_0 - g(x)] \tag{3.4}$$

$$\begin{aligned}(\mathcal{F}_2 x)(t) &= B^{-1}U(t,0)e(0,x(0)) - B^{-1}e(t,x(\sigma_1(t)) \\ &+ \int_0^t U(t,s)A(s)B^{-1}e(s,x(\sigma_1(s))ds \\ &+ \int_0^t U(t,s)B^{-1}\left[f(s,x(\sigma_2(s))) + \int_0^s k(s,\tau)h(\tau,x(\sigma_3(\tau)))d\tau\right]ds \\ &+ \sum_{0<t_i<t} B^{-1}U(t,t_k)I_k x(t_k), \quad \text{for all } t \in [0,a]. \end{aligned} \tag{3.5}$$

It is easy to see that the fixed point of $\mathcal{F}$ is the mild solutions of impulsive nonlocal problem $(2.1)-(2.3)$. Subsequently, we will prove that $\mathcal{F}$ has a fixed point by using Theorem 2.6.

First, we claim that the operator $\mathcal{F}$ is continuous on $\mathcal{PC}([0,a],X)$. For this purpose, we assume that $x_n \to x$ in $\mathcal{PC}([0,a],X)$. Then by $(M2-(ii))$ we get that

$$e(t,x_n(\sigma_1(t))) \to e(t,x(\sigma_1(t))), \quad a.e.\ t \in [0,a]$$

$$A(s)e(s,x_n(\sigma_1(s))) \to A(s)e(s,x(\sigma_1(s))), \quad a.e.\ \ s \in [0,a].$$

By the same reason $(M3-(ii))$ and $(M4-(ii))$ we get

$$f(s,x_n(\sigma_2(s))) \to f(s,x(\sigma_2(s))), \quad a.e.\ s \in [0,a]$$

$$h(\tau, x_n(\sigma_2(\tau))) \to h(\tau, x(\sigma_2(\tau))), \quad a.e.\ \tau \in [0,a].$$

Since $(M4-(ii)), (M5)$ hold, by the dominated convergence theorem, for every $s \in [0,a]$ we have

$$\int_0^s k(s,\tau)h(\tau, x_n(\sigma_3(\tau)))d\tau \to \int_0^s k(s,\tau)h(\tau, x(\sigma_3(\tau)))d\tau, \quad (n \to +\infty).$$

Thus

$$\begin{aligned}
\|\mathcal{F}x_n - \mathcal{F}x\| \leq\ & \alpha\beta M\|g(x_n) - g(x)\| + \alpha\|e(t, x_n(\sigma_1(t))) - e(t, x(\sigma_1(t)))\| \\
& +\alpha M \int_0^t \|A(s)e(s, x_n(\sigma_1(s))) - A(s)e(s, x(\sigma_1(s)))\|ds \\
& +\alpha M \int_0^t \|f(s, x_n(\sigma_2(s))) - f(s, x(\sigma_2(s)))\|ds \\
& +\alpha M \int_0^t \int_0^s \|k(s,\tau)h(\tau, x_n(\sigma_3(\tau)))d\tau - k(s,\tau)h(\tau, x(\sigma_3(\tau)))d\tau\|ds \\
& +\alpha M \sum_{0<t_i<t} \|I_k x_n(t_k) - I_k x(t_k)\| \\
\to\ & 0 \quad as \quad n \to \infty. \qquad (3.6)
\end{aligned}$$

That is $\mathcal{F}$ is continuous.

Next, we claim that the Monch's condition holds.
Suppose that $D \subseteq B_r$ is countable and $D \subseteq \overline{co}(0 \cup \mathcal{F}(D))$, we show that $\psi(D) = 0$, where B_r is the open ball of the radius r centered at the zero in $\mathcal{PC}([0,a], X)$. Without loss of generality, we may suppose that $D = \{x_n\}_{n=1}^{+\infty}$. By using the condition $(M1)-(M7)$, we can easily verify that $\{\mathcal{F}x_n\}_{n=1}^{+\infty}$ is equicontinuous. So, $D \subseteq \overline{co}(0 \cup \mathcal{F}(D))$ is also equicontinuous.

Now, from the Lemma 2.3, 2.4, 2.5 and the continuity of $B^{-1}U(t,0)B$, it follows that

$$\begin{aligned}
\psi(\{\mathcal{F}x_n\}_{n=1}^{+\infty}) \leq\ & \sup_{t\in[0,a]} \psi(\{B^{-1}U(t,0)Bg(x_n)\}_{n=1}^{+\infty}) + \psi(\{B^{-1}U(t,0)e(0,x(0))\}) \\
& +\psi\Big(\{B^{-1}e(t, x_n(\sigma_1(t)))\}_{n=1}^{+\infty}\Big) \\
& +\psi\Big(\{\int_0^t U(t,s)A(s)B^{-1}e(s, x_n(\sigma_1(s)))ds\}_{n=1}^{+\infty}\Big) \\
& +\psi\Big(\{\int_0^t U(t,s)B^{-1}f(s, x_n(\sigma_2(s)))ds\}_{n=1}^{+\infty}\Big) \\
& +\psi\Big(\{\int_0^t \int_0^s U(t,s)B^{-1}k(s,\tau)h(\tau, x_n(\sigma_3(\tau)))d\tau ds\}_{n=1}^{+\infty}\Big) \\
& +\psi\Big(\{\sum_{0<t_i<t} B^{-1}U(t,t_k)I_k x_n(t_k)\}_{n=1}^{+\infty}\Big) \\
\leq\ & 2\alpha M\gamma_0 + 2\alpha\gamma_e(t)\psi(\{x_n(\sigma_1(t))\}_{n=1}^{+\infty}) \\
& +2\alpha M \int_0^t \gamma_{Ae}(s)\psi(\{x_n(\sigma_1(s))\}_{n=1}^{+\infty})ds \\
& +2\alpha M \int_0^t \gamma_f(s)\psi(\{x_n(\sigma_1(s))\}_{n=1}^{+\infty})ds \\
& +2\alpha MK \int_0^t \int_0^s \gamma_h(s)\psi(\{x_n(\sigma_3(\tau))\}_{n=1}^{+\infty})d\tau ds
\end{aligned}$$

$$
\begin{aligned}
&+2\alpha M\gamma_I\psi(\{x_n(t_k)\}_{n=1}^{+\infty})\\
\leq\ & 2\alpha\Big[\|\gamma_e\|+M\{\|\gamma_0\|+\|\gamma_{Ae}\|+\|\gamma_f\|+K\|\gamma_h\|+\|\gamma_I\|\}\Big]\psi(\{x_n\}_{n=1}^{+\infty}).
\end{aligned}
$$

Thus, we get that

$$
\begin{aligned}
\psi(D) &\leq \psi(\overline{co}(0\cup\mathcal{F}(D)))\\
&= \psi(\mathcal{F}(D))\\
&\leq 2\alpha\Big[\|\gamma_e\|+M\{\|\gamma_0\|+\|\gamma_{Ae}\|+\|\gamma_f\|+K\|\gamma_h\|+\|\gamma_I\|\}\Big]\psi(D)
\end{aligned}
$$

which implies that $\psi(D)=0$, since the condition (3.2) holds.

Now let $\lambda\in(0,1)$ and $x=\lambda\mathcal{F}(x)$. Then, for $t\in[0,a]$

$$
\begin{aligned}
x(t) &= \lambda B^{-1}U(t,0)BE[x_0-g(x)]+\lambda B^{-1}U(t,0)e(0,x(0))\\
&\quad -\lambda B^{-1}e(t,x(\sigma_1(t))+\lambda\int_0^t U(t,s)A(s)B^{-1}e(s,x(\sigma_1(s))ds\\
&\quad +\lambda\int_0^t U(t,s)B^{-1}\left[f(s,x(\sigma_2(s))+\int_0^s k(s,\tau)h(\tau,x(\sigma_3(\tau)))d\tau\right]ds\\
&\quad +\lambda\sum_{0<t_i<t}B^{-1}U(t,t_k)I_kx(t_k)
\end{aligned}
$$

and one has

$$
\begin{aligned}
\|x(t)\| &\leq \alpha\beta M(\|x_0\|+c\|x\|+d)+\alpha M\phi_0+\alpha\phi_1(t)\Omega_e\|x\|\\
&\quad +\alpha M\int_0^a\phi_2(s)\Omega_{Ae}\|x\|ds+\alpha M\int_0^a\phi_3(s)\Omega_f\|x\|ds\\
&\quad +\alpha M\int_0^a K\phi_4(s)\Omega_h\|x\|ds+\alpha M\Omega_I\|x\|\\
&\leq \alpha\beta M(\|x_0\|+c\|x\|+d)+\alpha\|\phi_1\|_{\mathcal{L}^1}\Omega_e\|x\|\\
&\quad +\alpha M\Big[\|\phi_0\|+\|\phi_2\|_{\mathcal{L}^1}\Omega_{Ae}\|x\|+\|\phi_3\|_{\mathcal{L}^1}\Omega_f\|x\|+K\|\phi_4\|_{\mathcal{L}^1}\Omega_h\|x\|+\Omega_I\|x\|\Big].
\end{aligned}
$$

Consequently,

$$
\frac{(1-\alpha\beta Mc)\|x\|}{\alpha\beta M(d+\|x_0\|)+\alpha\phi_1\Omega_e\|x\|+\alpha M[\phi_0+\phi_2\Omega_{Ae}\|x\|+\phi_3\Omega_f\|x\|+K\phi_4\Omega_h\|x\|+\Omega_I\|x\|]}.
$$

Then by (3.1) there exists $\mathcal{N}$ such that $\|x\|\neq\mathcal{N}$. Set

$$
\mathcal{U}=\{x\in\mathcal{PC}([0,a],X):\|x\|<\mathcal{N}\}.
$$

From the choice of $\mathcal{U}$ there is no $x\in\partial U$ such that $x=\lambda\mathcal{F}(x)$, for some $\lambda\in(0,1)$. Thus we get a fixed point of $\mathcal{F}$ in $\overline{\mathcal{U}}$ due to Theorem 2.6, which is a mild solution to $(2.1)-(2.3)$. The proof is completed. □

Now, we will give the existence for $(2.1)-(2.3)$ when the nonlocal item g has no compactness. Assume the following holds:

(M8) The function $g:\mathcal{PC}([0,a],X)\to D(B)$ is Lipschitz continuous with constant $\mathcal{L}$.

Theorem 3.2. *Assume that the conditions $(M1)-(M6)$ and $(M8)$ are satisfied. Then for every $x_0\in D(B)$the impulsive nonlocal problem $(2.1)-(2.3)$ has at least one mild solution $[0,a]$ provided that there exists a constant $\mathcal{N}>0$ with*

$$
\frac{(1-\alpha\beta M\mathcal{L})\mathcal{N}}{\alpha\beta M(\|g(0)\|+\|x_0\|)+\alpha\phi_1\Omega_e(\mathcal{N})+\alpha M[\phi_0+\phi_2\Omega_{Ae}(\mathcal{N})+\phi_3\Omega_f(\mathcal{N})+K\phi_4\Omega_h(\mathcal{N})+\Omega_I(\mathcal{N})]}>1 \tag{3.7}
$$

and that

$$\alpha\beta M\mathcal{L}+2\alpha\Big[\|\gamma_e\|+M\{\|\gamma_0\|+\|\gamma_{Ae}\|+\|\gamma_f\|+K\|\gamma_h\|+\|\gamma_I\|\}\Big]<1. \tag{3.8}$$

Proof. On account of Theorem 3.1, we can prove that operator $\mathcal{F}$ defined by (3.3) is continuous on $\mathcal{PC}([0,a],X)$.

We prove that $\mathcal{F}$ satisfies the Monch's condition holds.

For this purpose, Let $D\subseteq B_r$ is countable and $D\subseteq \overline{co}(0\cup\mathcal{F}(D))$, we show that $\psi(D)=0$. Without loss of generality, we may suppose that $D=\{x_n\}_{n=1}^{+\infty}$. By using the condition $(\mathbf{A1})-(\mathbf{A3})$, we can easily verify that $\{\mathcal{F}_2x_n\}_{n=1}^{+\infty}$ is equicontinuous. Moreover, $\mathcal{F}_1:D\to\mathcal{PC}([0,a],X)$ is Lipschitz continuous with constant $\alpha\beta M\mathcal{L}$ due to the condition $(M8)$. In fact, for $x,y\in D$, we have

$$\begin{aligned}\|R_1x-R_1y\| &= \sup_{t\in[0,a]}\|B^{-1}U(t,0)Bg(x)-B^{-1}U(t,0)g(y)\|\\ &\le \alpha\beta M\|g(x)-g(y)\|\\ &\le \alpha\beta M\mathcal{L}\|x-y\|.\end{aligned}$$

So, from $(M2-(iii)),(M3-(iii)),(M4-(iii)),(M8)$ and Lemma 2.3, 2.4, 2.5 it follows that

$$\psi(\{\mathcal{F}x_n\}_{n=1}^{+\infty}) \le \psi(\{\mathcal{F}_1x_n\}_{n=1}^{+\infty})+\psi(\{\mathcal{F}_2x_n\}_{n=1}^{+\infty})$$

$$\begin{aligned}&\psi(\{\mathcal{F}x_n\}_{n=1}^{+\infty})\\ &\le \alpha\beta M\mathcal{L}\psi(\{x_n\}_{n=1}^{+\infty})+\sup_{t\in[0,a]}\psi(\{B^{-1}U(t,0)e(0,x(0))\})\\ &\quad+\psi\Big(\{B^{-1}e(t,x_n(\sigma_1(t)))\}_{n=1}^{+\infty}\Big)\\ &\quad+\psi\Big(\{\int_0^t U(t,s)A(s)B^{-1}e(s,x_n(\sigma_1(s)))ds\}_{n=1}^{+\infty}\Big)\\ &\quad+\psi\Big(\{\int_0^t U(t,s)B^{-1}f(s,x_n(\sigma_2(s)))ds\}_{n=1}^{+\infty}\Big)\\ &\quad+\psi\Big(\{\int_0^t\int_0^s U(t,s)B^{-1}k(s,\tau)h(\tau,x_n(\sigma_3(\tau)))d\tau ds\}_{n=1}^{+\infty}\Big)\\ &\quad+\psi\Big(\{\sum_{0<t_i<t}B^{-1}U(t,t_k)I_kx_n(t_k)\}_{n=1}^{+\infty}\Big)\\ &\le \alpha\beta M\mathcal{L}\psi(\{x_n\}_{n=1}^{+\infty})\\ &\quad+2\alpha\Big[\|\gamma_e\|+M\{\|\gamma_0\|+\|\gamma_{Ae}\|+\|\gamma_f\|+K\|\gamma_h\|+\|\gamma_I\|\}\Big]\psi(\{x_n\}_{n=1}^{+\infty})\\ &\le \Big\{\alpha\beta M\mathcal{L}+2\alpha[\|\gamma_e\|+M\{\|\gamma_0\|+\|\gamma_{Ae}\|+\|\gamma_f\|+K\|\gamma_h\|+\|\gamma_I\|\}]\Big\}\psi(\{x_n\}_{n=1}^{+\infty}).\end{aligned}$$

Thus, we get that

$$\begin{aligned}\psi(D) &\le \psi(\overline{co}(0\cup\mathcal{F}(D)))\\ &= \psi(\mathcal{F}(D))\\ &\le \alpha\Big\{\beta M\mathcal{L}+2[\|\gamma_e\|+M\{\|\gamma_0\|+\|\gamma_{Ae}\|+\|\gamma_f\|+K\|\gamma_h\|+\|\gamma_I\|\}]\Big\}\psi(D)\end{aligned}$$

which implies that $\psi(D)=0$, since the condition (3.8) holds.

Now with analogous arguments as in the proof of theorem 3.1, we can get an open ball U by the condition of (3.7), and there is no $x\in\partial\mathcal{U}$ such that $x=\lambda\mathcal{F}(x)$ for some $\lambda\in(0,1)$. Thus we get a fixed point of $\mathcal{F}$ in $\overline{\mathcal{U}}$ due to Theorem 2.3, which is a mild solution to $(2.1)-(2.3)$. The proof is completed. □

4. APPLICATION

The notion of controllability is of great importance in mathematical control theory. Many fundamental problems of control theory such as pole-assignment, stabilizability and optimal control may be solved under the assumption that the system is controllable. It means that it is possible to steer any initial state of the system to any final state in some finite time using an admissible control. During the last few decades, several authors have discussed the existence, uniqueness, and asymptotic behavior of the solution of these systems. Apart from these, the study of controllability and observability properties of a system in control theory is certainly, at present, one of the most active interdisciplinary areas of research. Control theory arises in most modern applications. On the other hand, control theory has remained a discipline where many mathematical ideas and methods have fused to produce a new body of important mathematics. In control theory, one of the most important qualitative aspects of a dynamical system is controllability. As far as the controllability problems associated with finite-dimensional systems modelled by ODEs are concerned, this theory has been extensively studied during the last decades. In the finite-dimensional context, a system is controllable if and only if the algebraic Kalman rank condition is satisfied. According to this property, when a system is controllable for some time, it is controllable for all the time. But this is no longer true in the context of infinite-dimensional systems modelled by PDEs.

As an application of Theorem 3.1 we shall consider the system $(2.1)-(2.3)$ with a control parameter such as

$$\begin{aligned}&\frac{d}{dt}\left[Bx(t)+e(t,x(\sigma_1(t)))\right]+A(t)x(t)\\&\quad=f(t,x(\sigma_2(t)))+Cu(t)+\int_0^t k(t,s)h(s,x(\sigma_3(s)))ds,\ t\neq t_k,\end{aligned}\tag{4.1}$$

$$x(0)+g(x) = x_0 \tag{4.2}$$

$$\Delta x(t_k) = I_k(x_{t_k}),\quad k=1,2,\dots,m \tag{4.3}$$

where A,B,f,g,e,h and I_k are as before and C is a bounded linear operator from a Banach space U into X and $u\in\mathcal{L}^2([0,a],U)$. The mild solution of $(4.1)-(4.3)$ is given by

$$\begin{aligned}x(t) &= B^{-1}U(t,0)B[x_0-g(x)]+B^{-1}U(t,0)e(0,x(0))\\&\quad -B^{-1}e(t,x(\sigma_1(t))+\int_0^t U(t,s)A(s)B^{-1}e(s,x(\sigma_1(s))ds\\&\quad +\int_0^t U(t,s)B^{-1}\left[f(s,x(\sigma_2(s)))+Cu(s)+\int_0^s k(s,\tau)h(\tau,x(\sigma_3(\tau)))d\tau\right]ds\\&\quad +\sum_{0<t_i<t}B^{-1}U(t,t_k)I_kx(t_k).\end{aligned}\tag{4.4}$$

Definition 5.1 ([7, 28]) System $(4.1)-(4.3)$ is said to be controllable on the interval $[0,a]$, if for every $x_0,\ x_1\in X$, there exists a control $u\in\mathcal{L}^2(I,U)$ such that the mild solution $u(\cdot)$ of $(4.1)-(4.3)$ satisfies $x(b)=x_1$.

To study the controllability result we need the following additional condition:

$(M8)$ The linear operator $W:\mathcal{L}^2(I,U)\to X$, defined by

$$Wu=\int_0^a B^{-1}U(a,s)Cu(s)ds$$

induces an inverse operator W^{-1} defined an $\mathcal{L}^2(I,V)/kerW$ and there exists a positive constant $\mathcal{M}_1 > 0$ such that $\|CW^{-1}\| \leq \mathcal{M}_1$.

Theorem 4.1. *If the assumptions* $(M1)-(M8)$ *are satisfied, then the system* $(4.1)-(4.3)$ *is controllable on* I.

Proof. Using the assumption $(M8)$, for an arbitrary function $u(\cdot)$, define the control

$$\begin{aligned} u(t) \quad = \quad & W^{-1}\Big[x_1 - B^{-1}U(t,0)B[x_0 - g(x)] + B^{-1}U(t,0)e(0,x(0)) \\ & -B^{-1}e(t,x(\sigma_1(t))) + \int_0^t U(t,s)A(s)B^{-1}e(s,x(\sigma_1(s))ds \\ & + \int_0^t U(t,s)B^{-1}\left[f(s,x(\sigma_2(s)))+Cu(s) + \int_0^s k(s,\tau)h(\tau,x(\sigma_3(\tau)))d\tau\right]ds \\ & + \sum_{0<t_i<t} B^{-1}U(t,t_k)I_k x(t_k)\Big](t). \end{aligned}$$

We shall show that when using this control, the operator $\mathcal{H} : Z \to Z$ defined by

$$\begin{aligned} & (\mathcal{H}u)(t) \\ & = \quad B^{-1}U(t,0)B[x_0 - g(x)] + B^{-1}U(t,0)e(0,x(0)) \\ & \quad -B^{-1}e(t,x(\sigma_1(t))) + \int_0^t U(t,s)A(s)B^{-1}e(s,x(\sigma_1(s))ds \\ & \quad + \int_0^t U(t,s)B^{-1}\left[f(s,x(\sigma_2(s))) + \int_0^s k(s,\tau)h(\tau,x(\sigma_3(\tau)))d\tau\right]ds \\ & \quad + \int_0^t U(t,s)B^{-1}CW^{-1}\Big[x_1 - B^{-1}U(t,0)B[x_0 - g(x)] + B^{-1}U(t,0)e(0,x(0)) \\ & \quad -B^{-1}e(t,x(\sigma_1(t))) + \int_0^a U(t,s)A(s)B^{-1}e(s,x(\sigma_1(s))ds \\ & \quad + \int_0^t U(t,s)B^{-1}\left[f(s,x(\sigma_2(s))) + Cu(s) + \int_0^s k(s,\tau)h(\tau,x(\sigma_3(\tau)))d\tau\right]ds \\ & \quad + \sum_{0<t_i<t} B^{-1}U(t,t_k)I_k x(t_k)\Big](t) + \sum_{0<t_i<t} B^{-1}S(t-t_k)I_k x(t_k) \end{aligned}$$

has a fixed point. This fixed point is, then a solution of $(4.1) - (4.3)$. Clearly, $(\mathcal{H}u)(a) = x(a) = x_1$, which means that the control u steers the system $(4.1)-(4.3)$ from the initial state x_0 to the final state x_1 at time a, provided we can obtain a fixed point of the nonlinear operator $\mathcal{H}$. The remaining part of the proof is similar to Theorem 3.1 and hence, it is omitted. □

5. EXAMPLE

Consider the partial integrodifferential equation of neutral type

$$\begin{aligned} & \frac{\partial}{\partial t}\Big[z(t,x) - z_{xx}(t,x) + \int_{-\infty}^t b(s-t)b_1(s,z(\sin s,x))ds\Big] \\ & = \; -a(t,x)\frac{\partial^2}{\partial y^2}z(t,x) + b_2(t,z(\sin t,x)) \\ & \quad + \frac{\sin z(t,x)}{(1+t)(1+t^2)}\int_0^t k(t,s)e^{-z(\sin s,x)}ds, \\ & \quad 0 \leq x \leq \pi, \quad t \in I \end{aligned} \tag{5.1}$$

$$z(t,0) = z(t,\pi) = 0, \quad t \geq 0 \tag{5.2}$$

$$z(0,x) + \sum_{i=1}^{m} e_i \phi_{t_i}(s,x) = z_0(x) \in \mathcal{PC}, \quad 0 \leq x \leq \pi \tag{5.3}$$

$$\Delta z|_{t=t_i} = I_i(z(x)) = (\gamma_i(z(x)) + t_i)^{-1}, \; z \in X, \; 1 \leq i \leq m, \tag{5.4}$$

where $a(t,x)$ is continuous on $0 \leq y \leq \pi, \; 0 \leq t \leq a$ and the constant e_i, γ_i are small and $b_1(t,s)$ is continuous such that $\|k(t,s)\| \leq K_B$. Let us take $X = U = L^2[0,\pi]$ endowed with the usual norm $|\cdot|_{L^2}$. Put $x(t) = z(t,x)$ is continuous norm $\|\cdot\|_{\mathcal{L}_2}$ and let

$$\begin{aligned}
e(t,\psi)(x) &= \int_0^{\pi} b_1(s-t)\psi(s,x)ds; \\
f(t,\psi)(x) &= b_2(t,\psi(\sin t, x)); \\
h(s,\psi)(x)ds &= \frac{\sin\psi(t,x)}{(1+t)(1+t^2)} \int_0^t e^{-\psi(\sin s,x)}ds; \\
I_i(\psi)(x) &= (\gamma_i|\psi(x)| + t_i)^{-1}; \\
g(\psi)(x) &= \sum_{i=1}^{m} e_i\phi_{t_i}(s,x).
\end{aligned}$$

Define the operator $A : \mathcal{D}(A) \subset X \longrightarrow X$ and $B : \mathcal{D}(B) \subset X \longrightarrow X$ by

$$Az = -z_{xx}, \quad Bz = z - z_{xx},$$

where each domain $\mathcal{D}(A)$ and $\mathcal{D}(B)$ is given by

$$\{z \in X : z, z_x \text{ are absolutely continuous, } z_{xx} \in X, z(0) = z(\pi) = 0\}.$$

Then A and B can be written, respectively, as

$$\begin{aligned}
Az &= \sum_{n=1}^{\infty} n^2 \langle z, z_n\rangle z_n, \; z \in \mathcal{D}(A) \\
Bz &= \sum_{n=1}^{\infty} (1+n^2)\langle z, z_n\rangle z_n, \; z \in \mathcal{D}(B),
\end{aligned}$$

where $z_n(x) = \sqrt{2/\pi}\sin(nx)$, $n = 1, 2, \ldots,$ is the orthogonal set of vectors of A. Furthermore for $z \in X$, we have

$$\begin{aligned}
B^{-1}z &= \sum_{n=1}^{\infty} \frac{1}{1+n^2}\langle z, z_n\rangle z_n; \\
-AB^{-1}z &= \sum_{n=1}^{\infty} \frac{-n^2}{1+n^2}\langle z, z_n\rangle z_n; \\
S(t)z &= \sum_{n=1}^{\infty} \exp\left(\frac{-n^2 t}{1+n^2}\right)\langle z, z_n\rangle z_n.
\end{aligned}$$

It is easy to see that AB^{-1} generates a strongly continuous semigroup $S(t)$ on Y and $S(t)$ is compact such that $|S(t)| \leq e^{-t}$ for each $t > 0$. Now we define the operator $A(t) : \mathcal{D}(A) \subset X \longrightarrow X$ by $A(t)z = a(t,x)Az(x)$. By assuming that $x \longrightarrow a(t,x)$ is continuous in t and there exist $\rho > 0$ such that $a(t,x) \leq -\rho$ for all $t \in I, \; x \in [0,\pi]$, it follows that the system

$$\begin{aligned}
z'(t) &= A(t)z(t), \quad t \geq s, \\
z(s) &= x \in X,
\end{aligned}$$

generates an evolution system $U(t,s)$ as $U(t,s)z = T(t-s)\exp(\int_s^t a(\tau,x)d\tau)z$, for $z \in X$ and $\|U(t,s)\| \leq e^{-(1+\rho)(t-s)}$, for every $t \geq s$.

With this choice of $A(t),\ e,\ f,\ h,\ g$ and I_i, we see that $(5.1)-(5.4)$ can be written in the abstract formulation of $(2.1)-(2.3)$. So all the conditions of the Theorem 3.1 are satisfied. Hence the equation $(5.1)-(5.4)$ has a mild solution.

References

[1] S. Aizicovici and H. Lee, Nonlinear nonlocal Cauchy problems in Banach spaces, Appl. Math. Lett., **18** (2005), 401-407.

[2] J. H. Ayerbe, T.D. Benavides and G. L. Acedo, Measure of noncmpactness in in metric fixed point theorem, Birkhauser, Basel., (1997).

[3] J. Banas and K. Goebel, Measure of Noncompactness in Banach Spaces, Lecture Notes in Pure and Appl. Math., **60**, Marcel Dekker, New York., (1980).

[4] J. Banas and W. G. El-Sayed, Measures of noncompactness and solvability of an integral equation in the class of functions of locally bounded variation, J. Math. Anal. Appl., **167**(1992), 133-151.

[5] K. Balachandran, Existence and uniqueness of mild and strong solutions of nonlinear integrodifferential equation with nonlocal condition, Diff. Equ. and Dyna. Syst., **6** (1998), 159-165.

[6] K. Balachandran and S. Ilamaran, Existence and uniqueness of mild and strong solutions of a Volterra integrodifferential equation with nonlocal conditions. Tamkang J. Math., **28**(1997), 93-100.

[7] K. Balachandran and J. Y. Park, Existence of solutions and controllability of nonlinear integrodifferential systems in Banach spaces, Math. Problems Engineering., **2** (2003), 65-79.

[8] K. Balachandran, J. Y. Park and M. Chandrasekaran, Nonlocal Cauchy problem for delay integrodifferential equations of Sobolev type in Banach spaces, Appl. Math. Let., **15**, (2002) 845-854.

[9] D. Bahuguna and R. Shukla, Approximation of Solutions to Nonlinear Sobolev type Evolution Equations, Electronic J. Differential Equations., **2003** (2003), 1-16.

[10] H. Brill, A semilinear Sobolev evolution equation in Banach space, J. Differential Equations., **24** (1977), 412-425.

[11] L. Byszewski, Theorems about the existence and uniqueness of solutions of a semilinear evolution nonlocal Cauchy problem, J. Math. Anal. Appl., **162** (1991), 494-505.

[12] L. Byszewski, Existence of approximate solution to abstract nonlocal Cauchy problem, J. Appl. Math. Stochastic Anal. **5**, (1992) 363-374

[13] L. Byszewski and V. Lakshmikantham, Theorem about the existence and uniqueness of solutions of a nonlocal Cauchy problem in a Banach space, Appl. Anal. **40** (1990), 11-19.

[14] F. S. De Blasi, On a property of the unit sphere in a Banach space, Bull. Math. Soc. Sci. Math. R.S. Roumanie., **21**, (1977) 259-262.

[15] X. Fu and K. Ezzinbi, Existence of solutions for neutral functional differential evolution equations with nonlocal conditions, Nonlinear Analysis. Theory, Methods & Applications. An International Multidisciplinary Journal. Series A: Theory and Methods. **54** (2003), 215-227.

[16] J. H. Hale and S. M. Verduyn Lunel, Introduction to Functional-Differential Equations, Springer-Verlag, New York., (1993).

[17] H. P. Heinz, On the behavior of measure of noncompactness with respect to differentiation and integration of vector-valued functions, Nonlinear Anal., **7** (1983), 1351-1371.

[18] E. Hernandez and H. R. Henriquez, Impulsive partial neutral differential equations, Applied Math. Let., **19** (2006), 215-222.

[19] M. E. Hernandez, Existence results for partial neutral functional differential equations with nonlocal conditions, Cadernos De Matematica., **2** (2001), 239-250.

[20] M. E. Hernandez and H. R. Henryquez, Existence results for partial neutral functional-differential equations with unbounded delay, J. Math. Anal. Appl., **221** (1998), 452-475.

[21] J. E. Lagnese, Exponential stability of solutions of differential equation of Sobolev type, SIAM J. Math. Anal., **3** (1972), 625-636.

[22] J. H. Lightboure III and S. M. Rankin III, A partial functional differential equation of Sobolev type, J. Math. Anal. Appl. **93** (1983), 328-337.

[23] V. Lakshmikantham, D. D. Bainov and P. S. Simeonov, Theory of Impulsive Differential Equations, World Scientific, Singapore, (1989).

[24] Y. Lin and J. H. Liu, Semilinear integrodifferential equations with nonlocal Cauchy problem, J. Integral Equa. Appl., **15** (2003), 79-93.

[25] M. Kamenskii, V. Obukhovskii and P. Zecca, Condensing multivalued maps and semilinear differential inclusions in Banach spaces, De Gruyter Series. Nonlinear Anal. Appl., **7** de Gruyter, Berlin., (2001).

[26] J. Liang, J. H. Liu and T. J. Xiao, Nonlocal impulsive problems for nonlinear differential equations in Banach spaces, Math. Computer Modelling., **49** (2009), 798-804.

[27] A. Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations, Springer-Verlag, New York., (1983).

[28] B. Radhakrishnan and K. Balachandran, Controllability of neutral evolution integrodifferential systems with state dependent delay, J. Optim Theory Appl., **153** (2012), 85-97.

[29] A. M. Samoilenko and N. A. Perestyuk, Impulsive Differential Equations, World Scientific, Singapore., (1995).

[30] R. E. Showalter, Existence and representation theorem for a semilinear Sobolev equation in Banach space, SIAM J. Math. Anal., **3** (1972), 527-543.

[31] X. Xue, Nonlocal nonlinear differential equations with measure of noncompactness in Banach spaces, Nonlinear Anal.: Theory Methods Appl., **70** (2009), 2593-2601.