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CONTINUITY OF FUZZY TRANSITIVE ORDERED SETS

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ABSTRACT. In the present paper we introduce and study the continuity for a set equipped with a transitive fuzzy binary order relation which we call a f-toset. Our work is inspired by the slogan: " Order theory is the study of transitive relations" due to S. Abramsky and A. Jung [1].

KEYWORDS : Fuzzy set; Fuzzy order relation; Continuous lattices; Continuous posets.
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1. INTRODUCTION

In crisp setting , S. Abramsky and A. Jung [1] introduced a method to construct canonical partially ordered set from a pre-ordered set and said: " Many notions from theory of partially ordered sets make sense even if reflexivity fails". Finally they sum up these considerations with the slogan: "Order theory is the study of transitive relations". In our opinion this slogan still valid in fuzzy setting . Thus From this point of view the present paper is devoted to introduce and study the continuity for a set with a transitive fuzzy binary order relation (so called a f-toset).

It is worth to mention that in crisp setting the continuous lattices were studied in [7] and types of continues posets (domains) were studied in [1, 7, 8, 10, 15].

Recently [13], the concept of continuity of some types of fuzzy directed complete posets was studied.

This paper consists of 3 Sections. In Section 2, some preliminaries and some basic concepts on f-toset are discussed. Section 3, is devoted to introduce and study the concept of continuous f-toset. Finally, a conclusion is given to compare some types of fuzzy posets.

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2. PRELIMINARIES AND SOME BASIC CONCEPTS

In this section we introduce the concept of fuzzy transitive ordered set and some of its basic concepts.

In this paper we use Claude Ponsard's definition of fuzzy order relation (see[2]).

Definition 2.1. Let X be a crisp set . A fuzzy order relation on X is a fuzzy subset of $X \times X$ satisfying the following three properties:

(i) $\forall\ x \in X,\ r(x,x) \in [0,1]$;

(ii) $\forall\ x,y \in X,\ r(x,y) + r(y,x) > 1$ implies $x = y$;

(iii) $\forall\ x,y,z \in X,\ r(x,y) \geq r(y,x)$ and $r(y,z) \geq r(z,y)$ implying $r(x,z) \geq r(z,x)$.

The pair (x,y) is called a fuzzy ordered set.

Definition 2.2. Let X be a crisp set . A fuzzy transitive order relation r on X is a fuzzy set of $X \times X$ satisfying (iii) in Definition 2.1. The pair (X,r) is called a fuzzy transitive order set (in brief f-toset).

Definition 2.3. Let (X,r) be a f-toset and let $A \subseteq X$. Then :

(1) The lower (resp. upper) bounded subset in X of A is denoted by $lb(A)$ (resp. $ub(A)$) and defined as follows :

$lb(A) = \{x \in X : \forall\ y \in A,\ r(y,x) \leq r(x,y)\}$ (resp. $ub(A) = \{x \in X : \forall\ y \in A,\ r(x,y) \leq r(y,x)\}$). Each element in $lb(A)$ (resp. $ub(A)$) is called a lower (resp. an upper) bound of A;

(2) The subset of least (resp. largest) elements of A is denoted by $le(A)$ (resp. $la(A)$) and defined as follows :

$le(A) = \{x \in A : \forall\ y \in A,\ r(y,x) \leq r(x,y)\}$ (resp. $la(A) = \{x \in A : \forall\ y \in A,\ r(x,y) \leq r(y,x)\}$). Each element in is called a least (resp. largest) element of A;

(3) The infimum (resp. supremum) subset in X of A is denoted by $\inf(A)$ (resp. $\sup(A)$) and defined as follows:

$\inf(A) = la(lb(A))$ (resp. $\sup(A) = le(ub(A))$). Each element in inf(A) (resp. sup(A)) is called an infimum (resp. a supremum) of A;

(4) The lower (resp. upper) closure in X of A is denoted by $\downarrow (A)$ (resp. $\uparrow (A)$) and defined as follows:

$\downarrow (A) = \{x \in X : \exists\ y \in A\ s.t.\ r(y,x) \leq r(x,y)\}$ (resp. $\uparrow (A) = \{x \in X : \forall\ y \in A\ s.t.\ r(x,y) \leq r(y,x)\}$).

Remark 2.3. In any f-toset (X,r) one can remark that for any subset A of X, $la(A)$, $le(A)$, $\sup(A)$ and $\inf(A)$ need not be singletons.

Remark 2.4. In [8], the author considered the supremum (resp. infimum) of a subset A of fuzzy ordered set (X,r) as the unique least element (resp. unique largest element) of the set of upper bounds (resp. lower bounds) of A if it exists.

Now we introduce some propositions on the fuzzy lower and fuzzy upper closure of a subset in a f-toset without proof.

Proposition 2.1. Let (X,r) be a f-toset and let $A,B \subseteq X$. Then:

(1) $\downarrow (\phi) = \phi$ and $\downarrow (X) \subseteq X$;

(2) $\uparrow (\phi) = \phi$ and $\uparrow (X) \subseteq X$;

(3) $If\ A \subseteq B$ then $\downarrow (A) \subseteq \downarrow (B)$;

(4) $If\ A \subseteq B$ then $\uparrow (A) \subseteq \uparrow (B)$;

(5) $\downarrow\downarrow (A) \subseteq \downarrow (B)$;

(6) $\uparrow\uparrow (A) \subseteq \uparrow (B)$;

(7) $If\ A \subseteq \downarrow (B)$ then $\downarrow (A) \subseteq \downarrow (B)$;

(8) $If\ A \subseteq \uparrow (B)$ then $\uparrow (A) \subseteq \uparrow (B)$.

Proposition 2.2. Let $(X, \leq)$ be a f-toset and let $\{A_j : j \in J\}$ be a family of sub-sets of X. Then:

(1) $\downarrow (\cup_{j\in J} A_j) = \cup_{j\in J} \downarrow (A_j)$;
(2) $\uparrow (\cup_{j\in J} A_j) = \cup_{j\in J} \uparrow (A_j)$;
(3) $\uparrow (\cap_{j\in J} A_j) = \cap_{j\in J} \uparrow (A_j)$; and
(4) $\downarrow (\cap_{j\in J} A_j) = \cap_{j\in J} \downarrow (A_j)$.

Definition 2.3. Let $(X, \leq)$ be a f-toset and let $A, B \subseteq X$. A is called:

(1) a directed (resp. filtered) subset iff $A \neq \phi$ and for every distinct points x, y in A, $\exists\ z \in A \cap\ ub(\{x, y\})$ (resp. $z \in A \cap\ lb(\{x, y\})$);

(2) a cofinal in B iff $A \subseteq B \subseteq \downarrow (A)$.

Proposition 2.3. Let $(X, \leq)$ be a f-toset and let $A, B \subseteq X$. If B is directed subset and cofinal in A, then A is directed subset and $sup(A) = sup(B)$.

Proof. First, we prove that A is a directed subset. Since $B \subseteq A$, then $A \neq \phi$. Let $l, m \in A$ s.t. $l \neq m$. Then $\exists\ b_1, b_2 \in B$ s.t. $r(b_1, l) \leq r(l, b_1), r(b_2, m) \leq r(m, b_2)$ and $b \in ub(\{b_1, b_2\}) \cap A$. Hence A is a directed subset.

Second, one can deduce that $ub(A) = ub(B)$ (Indeed, since $B \subseteq A$, then $ub(A) \subseteq ub(B)$. Conversely, $y \notin ub(A) \Rightarrow \exists\ a \in A$ s.t. $r(y, a) \nleq r(a, y) \Rightarrow \exists\ a \in \downarrow (B)$ s.t. $r(y, a) \nleq r(a, y) \Rightarrow \exists\ b \in B$ s.t. $r(b, a) \leq r(a, b)$ and $r(y, a) \nleq r(a, y) \Rightarrow\ b \in B$ s.t. $r(y, b) \nleq r(b, y) \Rightarrow y \notin ub(B)$. Hence $ub(B) \subseteq ub(A)$.). Thus sup(A) = sup(B).

The concept of way below relation is extended in f-toset as follows:

Definition 2.4. Let (X, r) be a f-toset and let $x, y \in X$, we say x is way below (resp. y is way above) y (resp. x), written $x \ll y$ iff for every directed subset D of X if $y \in \downarrow (sup(D))$, there exists $d \in D$ s.t. $r(d, x) \leq r(x, d)$. The family of the elements in X each of which way above (resp. way below) x is denoted and defined as follows:

$\Uparrow x = \{y \in X : x \ll y\}$ (resp. $\Downarrow x = \{y \in X : y \ll x\}$).

Proposition 2.4. In f-toset $(X, \leq)$ let $x, y, z \in X$. Then:

(1) If $r(y, x) \leq r(x, y)$ and $y \ll z$, then $x \ll z$;
(2) If $x \ll y$ and $r(z, y) \leq r(y, z)$, then $x \ll z$;
(3) If $\sup(\{y\}) \neq \phi$ and $x \ll y$, then $r(y, x) \leq r(x, y)$; and
(4) If $\sup(\{y\}) \neq \phi$ or $\sup(\{z\}) \neq \phi$, $x \ll y$ and $y \ll z$, then $x \ll z$.

Proof. (1) Let D be a directed subset of X s.t. $z \in \downarrow (\sup(D))$. Then $\exists\ d \in D$ s.t. $r(d, y) \leq r(y, d)$. Then $r(d, x) \leq r(x, d)$ and hence $x \ll z$.

(2) Let D be a directed subset of X s.t. $z \in \downarrow (\sup(D))$. Then $\exists\ k \in \sup(D)$ s.t. $r(k, z) \leq r(z, k)$. Thus $r(k, y) \leq r(y, k)$ and so $y \in \downarrow (\sup(D))$. Therefore $\exists\ l \in D$ s.t. $r(l, x) \leq r(x, l)$. Hence $x \ll z$.

(3) Let $D = \{y\}$ and assume that $x \ll y$. Then $\exists\ d \in D$ s.t. $r(d, x) \leq r(x, d)$ but $y = d$. Thus $r(y, x) \leq r(x, y)$.

(4) From (1) -(3) above we can prove (4).

The domain f-toset is defined as follows:

Definition 2.5. A f-toset (X, r) is called a domain f-toset iff for every directed subset A of X, $\sup(A) \neq \phi$.

3. CONTINUOUS F-TOSETS

First we introduce the following needed lammas without proof.

Lemma 3.1. Let (X, r) be a f-toset. If $\forall\ x \in X$, $\Downarrow x$ is a directed subset of X, then $\forall\ z \in X$, $D = \cup\{\Downarrow a : a \in \Downarrow z\}$ is a directed subset.

Lemma 3.2. Let (X, r) be a f-toset and let $x \in X$. Then $\forall\ x \in X$, $ub(\cup\{\Downarrow a : a \in \Downarrow$

$x\}) = ub(\cup\{\sup(\Downarrow a) : a \in\Downarrow x\})$. Thus $\sup(\cup\{\Downarrow a : a \in\Downarrow x\}) = \sup(\cup\{\sup(\Downarrow a) : a \in\Downarrow x\})$.

Now , we present the concept of continuity of a f-toset.

Definition 3.1. Let $(X, \leq)$ be a f-toset. It is said to be a continuous f-toset iff the following conditions are satisfied:
(1) $\sup(\{x\}) \neq \phi$;
(2) $\Downarrow x$ is a directed subset of X; and
(3) $x \in\downarrow (\sup(\cup\{\sup(\Downarrow a) : a \in\Downarrow x\}))$.

Theorem 3.1 (Interpolation). If (X, r) is a continuous f-toset, then the way below relation $'\ll'$ is interpolative, i.e., $x, z \in X$, $x \ll z$ implies that $\exists\ y \in X$ s.t. $x \ll y \ll z$.

Proof. From Lemmas 3.1 and 3.2 , $z \in\downarrow (\sup(\cup\{\Downarrow y : y \in\Downarrow z\}))$ and $\cup\{\Downarrow y : y \in\Downarrow z\}$ is directed. Then $\exists\ d \in\Downarrow y$ for some $y \in\Downarrow z$ s.t. $r(d, x) \leq r(x, d)$. From Proposition 2.4(1), we have that $x \ll y$. Hence $x \ll y \ll z$.

From Proposition 2.4(4) and Theorem 3.1 one can have the following result concerning with continuous information system . For the definition of continuous information system see [9].

Theorem 3.2. If (X, r), is a continuous f-toset, then $(X, \ll)$ is a continuous information system.

Lemma 3.3. For any f-toset (X, r), if the conditions
$(A)\ \forall\ x \in X,\ \sup(\{x\}) \neq \phi$ and
$(B)\ '\ll'$ is interpolative are satisfied,
then $\forall\ x \in X,\ \Downarrow x = \cup\{\Downarrow a : a \in\Downarrow x\}$.

Proof. First, let $z \in \cup\{\Downarrow a : a \in\Downarrow x\}$. Then $\exists\ a \in \Downarrow x$ s.t. $z \ll a$. From Proposition 2.4(4), $z \ll x$, i.e., $z \in\Downarrow x$. Second, let . Then $z \ll x$. Since $'\ll'$ is interpolative, then $\exists\ a \in X$ s.t. $z \ll a \ll x$, i.e., $z \in \cup\{\Downarrow a : a \in\Downarrow x\}$.

Applying Lemma 3.2, Lemma 3.3, Theorem 3.1 and Theorem 3.2 we introduce the following characterization of continuous f-tosets.

Theorem 3.3. (X, r) is a continuous f-toset iff the following conditions are satisfied:
(1) $\forall\ x \in X,\ \sup(\{x\}) \neq \phi$;
(2) $\forall x \in X, \Downarrow x$ is directed;
(3) $\ll$ is interpolative ; and
(4) $\forall\ x \in X,\ x \in\downarrow (\sup(\Downarrow x))$.

Proof. First of all, we note that conditions (1) and (2) above are common.
$\underline{\Rightarrow}$: From Theorem 3.1, $'\ll'$ is interpolative so that Condition (3) above is satisfied . From Lemma 3.2 and Lemma 3.3, Condition (4) above is satisfied.
$\underline{\Leftarrow}$: From Lemma 3.3, one can have that Condition (3) in Definition 3.1 is satisfied.

In the following we add more characterizations of continuous f-tosets

Theorem 3.4. (X, r) is a continuous f-toset iff the following conditions are satisfied:
(1) $\forall\ x \in X, \sup(\{x\}) \neq \phi$;
(2) $\ll$ is interpolative ; and
(3) $\forall\ x \in X, \exists$ a directed subset D of $\Downarrow x$ s.t. $x \in\downarrow (\sup(D))$.

Proof. $\underline{\Rightarrow}$: From Theorem 3.3, Conditions (1) and (2) above are satisfied. Condition (3) is satisfied if we put $D = \Downarrow x$.
$\underline{\Leftarrow}$: Now Conditions (1) and (3) in Theorem 3.3 are given in Theorem 3.4 as (1) and (2) above. We need to prove that D is cofinal in $\Downarrow x$. First $D \subseteq\Downarrow x$ and D is directed. Second, let $y \in\Downarrow x$. Since $x \in\downarrow (\sup(D))$, then $\exists\ d \in D$ s.t. $r(d, y) \leq r(y, d)$. So, $y \in\downarrow (D)$. Then from Proposition 2.3, $\Downarrow x$ is directed and $\sup(\Downarrow x) = \sup(D)$. Hence Conditions (2) and (4) in Theorem 3.3 are satisfied.

Theorem 3.5. (X, r) is a continuous f-toset iff the following conditions are satisfied:

(1) $\forall\, x \in X, \sup(\{x\}) \neq \phi$; and

(2) $\forall\, x \in X, \exists$ a directed subset D of $\cup\{\Downarrow\ a : a \in\Downarrow x\}$ s.t. $x \in\downarrow (\sup(D))$.

Proof. $\underline{\Rightarrow}$: From Theorem 3.4 and Lemma 3.3 one can have that $\Downarrow x = \cup\{\Downarrow a : a \in\Downarrow x\}$ so that from Condition (3) in Theorem 3.4 one have directly Condition (2) in above.

$\underline{\Leftarrow}$: Since D is cofinal in $\cup\{\Downarrow a : a \in\Downarrow x\}$ (Indeed, $D \subseteq \cup\{\Downarrow a : a \in\Downarrow x\}$. Let $z \in \cup\{\Downarrow a : a \in\Downarrow x\}$. Then $z \ll a$ for some $a \in\Downarrow x$ so that from Proposition 2.4(4) $z \ll x$. Since D is directed and $x \in\downarrow (sup(D))$, then $\exists\, d \in D$ s.t. $r(d,z) \leq r(z,d)$, i.e., $z \in\downarrow (D)$.), then from Proposition 2.3, $\cup\{\Downarrow a : a \in\Downarrow x\}$ is directed and $\sup(D) = \sup(\cup\{\Downarrow a : a \in\Downarrow x\})$. Hence condition (3) in the Theorem 3.4 is satisfied. Also one can prove that $'\ll'$ is interpolative (Indeed, let $x \ll z$ and from Condition (2) above $z \in\downarrow (\sup(\cup\{\Downarrow a : a \in\Downarrow z\}))$. Thus $\exists\, d \in \cup\{\Downarrow a : a \in\Downarrow z\}$ s.t. $r(d,x) \leq r(x,d)$ and $d \ll a \ll z$. So , from Proposition 2.4(1), $x \ll a$. Then $x \ll a \ll z$.). Then Condition (2) in Theorem 3.4 is satisfied. Hence (X,r) is a continuous f-toset .

Remark 3.1. From Lemma 3.1, one can write $\Downarrow x$ in Theorem 3.5 instead of $\cup\{\Downarrow a : a \in\Downarrow x\}$.

The concept of a base for a f-toset is introduced as follows:

Definition 3.2. Let $(X,\ll)$ be a f-toset. A subset B of X is called a base for X iff the following conditions are satisfied:

(1) $\forall x \in X, \sup(\{x\}) \neq \phi$; and

(2) $\forall x \in X$, $\exists$ a directed subset D of B s.t. $D \subseteq \cup\{\Downarrow a : a \in\Downarrow x\}$ and $x \in\downarrow (\sup(D))$.

Finally, we ‘give a characterization of continuous f-toset via the concept of the base of a f-toset.

Theorem 3.6. $(X,\ll)$ is a continuous f-toset iff it has a base.

Proof. $\underline{\Rightarrow}$: From Theorem 3.5, put $B = \cup_{x\in X}(\cup\{\Downarrow a : a \in\Downarrow x\})$.

$\underline{\Leftarrow}$: Condition (2) in Theorem 3.5 is satisfied directly from the Definition of the base of a f-toset.

Conclusion. (1) Recently [13], the concept of continuity of some types of fuzzy directed complete posets was studied. In Wei Yao's paper [13] he proved the equivalence between the fuzzy partial order in the sense of Bělohlăvek [2, 3] and the fuzzy order in the sense of Fan and Zhang [6, 14, 16]. In the present paper we study above the continuity of fuzzy partial poset due to Claude Ponsard [4] with regard S. Abramsky and A. Jung's slogan [1].

(2) First we recall the definition of Fan and Zhang [6, 14, 16] for $L = [0,1]$ and $* = \wedge = \min$.

Definition [6, 14, 16]**.** A Fan-Zhang-fuzzy partial order on a set X is function $e : X \times X \longrightarrow [0,1]$ satisfying

(a) $\forall\, x \in X,\ e(x,x) = 1$,

(b) $\forall\, x,y,z \in X,\ e(x,y) \wedge e(y,z) \leq e(x,z)$,

(c) $\forall\, x,y \in X,\ e(x,y) = e(y,x) = 1$ implies $x = y$.

(3) The following counterexamples illustrate that the concept of fuzzy partial order in the sense of Fan and Zhang[6, 14, 16] and the concept of fuzzy order in the sense of Cloude Ponsard [4] are independent notions.

Counterexample 1. Let $X = \{x,y,z\}$ and $R_1 : X \times X \longrightarrow [0,1]$ defined as follows: $R_1(x,x) = R_1(y,y) = R_1(z,z) = R_1(y,x) = R_1(x,z) = R_1(z,x) = R_1(z,y) = \frac{1}{4}$ and $R_1(x,y) = R_1(y,z) = \frac{1}{2}$. Since $\frac{1}{4} = R_1(x,z) \ngeq R_1(x,y) \wedge R_1(y,z) = \frac{1}{2}$, then R_1 is not fuzzy partial order in the sense of Fan and Zhang. One can check that R_1 is a fuzzy partial order in the sense of Claude Ponsard [4] (Remark that

$\frac{1}{2} = R_1(x,y) \geq R_1(y,x) = \frac{1}{4}$ and $\frac{1}{2} = R_1(y,z) \geq R_1(z,y) = \frac{1}{4}$ implies $\frac{1}{4} = R_1(x,z) \geq R_1(z,x) = \frac{1}{4}$.).

Counterexample 2. Let $X = \{x, y, z\}$ and $R_2 : X \times X \longrightarrow [0,1]$ defined as follows: $R_2(x,x) = R_2(y,y) = R_2(z,z) = 1$, $R_2(x,y) = R_2(y,x) = R_2(x,z) = R_2(z,x) = R_2(z,y) = \frac{1}{4}$ and $R_2(y,z) = \frac{1}{2}$. Since $R_2(x,y) \geq R_2(x,z) \wedge R_2(z,y)$, $R_2(y,x) \geq R_2(y,z) \wedge R_2(z,x)$, $R_2(x,z) \geq R_2(x,y) \wedge R_2(y,z)$, $R_2(z,x) \geq R_2(z,y) \wedge R_2(y,x)$, $R_2(z,y) \geq R_2(z,x) \wedge R_2(x,y)$, $R_2(y,z) \geq R_2(y,x) \wedge R_2(x,z)$. Then one can observe that R_2 is a fuzzy partial order in the sense of Fan and Zhang. Since,

then R_1 is not fuzzy partial order in the sense of Fan and Zhang. One can check that R_1 is a fuzzy partial order in the sense of Claude Ponsard [4] (Remark that $\frac{1}{2} = R_1(x,y) \geq R_1(y,x) = \frac{1}{4}$ and $\frac{1}{4} = R_1(y,z) \geq R_1(z,y) = \frac{1}{4}$ implies $\frac{1}{4} = R_1(x,z) \geq R_1(z,x) = \frac{1}{4}$.). $\frac{1}{4} = R_2(x,y) \geq R_2(y,x) = \frac{1}{4}$ and $\frac{1}{4} = R_2(z,x) \geq R_2(x,z) = \frac{1}{4}$ but $\frac{1}{4} = R_2(z,y) \ngeq R_2(y,z) = \frac{1}{2}$, then R_2 is not fuzzy partial order in the sense of Claude Ponsard.

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